

**Homework 4**  
**Math 302 - Fall 2006**  
**Prof. John A. Pelesko**

This homework is designed as practice for your first exam. You should be able to complete this homework in 50 minutes without access to your notes or text book.

(1) (15 points) Find all *equilibrium* solutions of

$$\frac{dy}{dt} = y - y^3$$

and determine the stability of each solution.

(2) (20 points) Solve the initial value problem

$$\frac{dy}{dt} = y - y^3, \quad y(0) = 1/2.$$

What is the limit of  $y(t)$  as  $t$  tends to infinity?

(3) (25 points) A tank contains 50 liters of water. Salt water containing  $c$  grams per liter of salt enters the tank at the rate of 1.5 liters per minute. An open drain at the bottom of the tank allows the solution to leave at the rate of 1 liter per minute. If, after 20 minutes, you want the concentration of salt in the tank to be 20 grams per liter, what value should you choose for  $c$ ?

(4) (20 points) Solve

$$x \frac{dy}{dx} - y = 2x^2y$$
$$y(1) = 1$$

(5) (20 points) Solve

$$\sin(x) \frac{dy}{dx} + 2 \cos(x)y = \sin(x) \cos(x)$$

(Hint: Think log's.)