

**Assignment 2**  
**Math 302 - Fall 2006**  
**Prof. J. A. Pelesko**  
**Due September 18, 2006**

(1) (25 points) In class, we derived the equation that allows us to best fit your data for the pipe/water experiment to your theory.

(a) Solve this equation to determine  $\beta$ .

(b) Using this value of  $\beta$  and other known values of parameters, compute  $\alpha$ .

(c) Carefully make a well-labelled plot comparing your data to your best fit theory.

(d) Using the value of  $\alpha$  above, predict the time it would have taken the tube to empty if the hole at the bottom had been two centimeters in diameter.

(2) (25 points) Assume that the air pressure  $p$  at an altitude  $h$  above sea level is proportional to the mass of the column of air above a horizontal unit area at that altitude, and also that the product of the volume of a given mass of air and the pressure on it remains constant at all altitudes. If  $p = p_0$  at sea level, find  $p$  as a function of  $h$ . Plot your result.

(3) (10 points) Solve

$$x \frac{dy}{dx} = x^5 + x^3 y^2 + y$$

(4) (10 points) Solve

$$y' + \cot(x)y = 2x \csc(x)$$

(5) (10 points) The equation

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

which is known as Bernoulli's equation is linear when  $n = 0$  or  $1$ . Show that it can be reduced to a linear equation for any other value of  $n$  by the change of variable  $z = y^{1-n}$ .

(6) (10 points) Apply the method of Problem (5) to solving

$$xy' + y = x^4 y^3$$

(7) (10 points) A curve rises from the origin in the  $x - y$  plane into the first quadrant. The area under the curve from  $(0, 0)$  to  $(x, y)$  is one-third the area of the rectangle with these points as opposite vertices. Find the equation of the curve.