Exam #1
Math 243

This is exam #1 of the Spring 2004 semester. Except for problem #1 you must show all of your work and be sure to clearly indicate your final answer. Good luck!

(1) (15 points) Which of the following surfaces most closely resembles a nuclear power plant cooling tower? (You don't need to show your work.)

(a) $-3x^2 - 2y^2 + z^2 = 1$ (b) $2z^2 = 4x^2 + 4y^2$

(c) $\frac{x^2}{4} + \frac{y^2}{4} - \frac{z^2}{10} = 1$ (d) $4x^2 + 4y^2 - 10z^2 = 1$

Plot using Maple, the answer is (c). (Use implicitplot3d.)

(2) (15 points) Consider the vectors $\vec{v}_1 =< 1, 2, a >$ and $\vec{v}_2 =< 2a, 1, 3 >$. Find a value of a that makes v_1 and v_2 orthogonal. Find a vector perpendicular to these two vectors.

The vectors are orthogonal iff the dot product is orthogonal, the dot product is

$$\vec{v}_1 \cdot \vec{v}_2 = 5a + 2 \tag{1}$$

hence $a = -2/5$.

To find a vector perpendicular to these two take the cross product to find

$$\vec{v}_1 \times \vec{v}_2 =< 32/5, -67/25, 13/5 > \tag{2}$$

(3) (15 points) Which two of the four parameterized lines below represent the same line?

$$\begin{aligned}
\vec{r}_1(t) &= <2,1,0> + <1,-\frac{1}{2},4> t \\
\vec{r}_2(t) &= <0,1,2> + <2,-1,8> t \\
\vec{r}_3(t) &= <4,0,8> + <2,-1,-8> t \\
\vec{r}_4(t) &= <2,0,1> + <-4,2,16> t
\end{aligned}$$

To test, see which line are parallel, i.e., you find $\vec{r}_1$ and $\vec{r}_3$ and $\vec{r}_2$ and $\vec{r}_4$ are parallel. Next, test to see if they have any points in common, they do not, hence none of the lines are the same.

(4) (20 points) Find the area of the triangle with vertices $(-3,-2,-2)$,$(2,3,-2)$, and $(1,-2,2)$. The triangle is half of a parallelogram. Use the cross product, i.e., let

$$\vec{AB} = < -5, -5, 0 > \tag{3}$$

$$\vec{AC} = < -4, 0, -4 > \tag{4}$$

and then

$$\text{Area} = (1/2)|\vec{AB} \times \vec{AC}| = 10\sqrt{3} \tag{5}$$

(5) (25 points) Identify the surface(s) given by each of the following equations. Show your work, that is reduce to normal form as needed.

(a) $z = 4x^2 + 20y^2$

(b) $z = 4r^2(\cos^2(\theta) + 5\sin^2(\theta))$

(c) $\rho = 7$

(d) $x^2 - y^2 + 4y + z = 4$

(e) $x + y + z = c$

(a) is a paraboloid, (b) is a paraboloid, (c) is a sphere, (d) is a hyperboloid, and (e) a plane.