M221 EXAM III

Name (Please Print) KEY

SHOW ALL WORK FOR FULL CREDIT

There are 12 questions, each worth 8 points unless indicated otherwise.

1. a) Let $h(x)$ be any differentiable function and $h(x) = e^{x^2+x}$. Determine $h'(x)$.

$$h'(x) = e^{x^2+x}(2x+1) = \underline{\underline{(2x+1)e^{x^2+x}}}$$

b) Let $f(x)$ be any differentiable function and $f(x) = \ln\left(\frac{x}{8-x}\right)$. Determine $f'(x)$.

$$f'(x) = \frac{1}{\frac{x}{8-x}}\left[\frac{1(8-x)-x(-1)}{(8-x)^2}\right]$$

$$f'(x) = \frac{(8-x)}{x}\left[\frac{8-x+x}{(8-x)^2}\right]$$

$$f'(x) = \frac{(8-x)}{x}\left[\frac{8}{(8-x)^2}\right] = \underline{\underline{\frac{8}{x(8-x)}}}$$

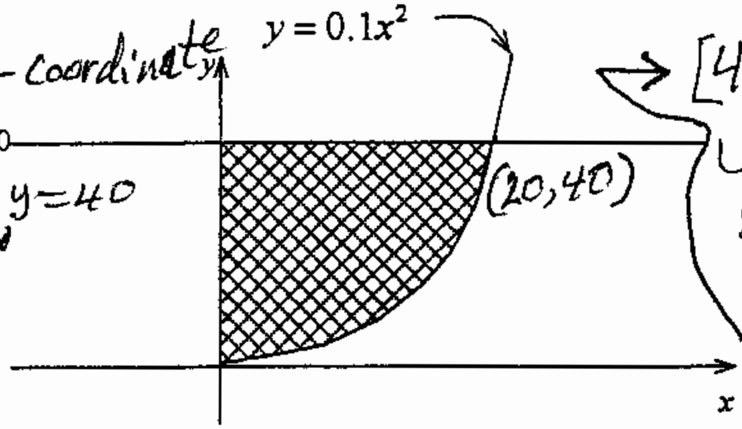
2. Find the area of the shaded region in the figure below bounded by $y = 0.1x^2$ and $y = 40$. **[9 pts]**

(i) Get the x-coordinate where $y=0.1x^2$ and $y=40$ intersect:

$0.1x^2 = 40$

$x^2 = 400$

$x = \pm 20$



(ii) $\displaystyle\int_0^{20}(40-0.1x^2)\,dx = \left(40x - \frac{0.1x^3}{3}\right)\Big|_0^{20}$

$\rightarrow \left[40(20) - \frac{0.1(20)^3}{3}\right] - \underbrace{\left[40(0) - \frac{.1(0)^3}{3}\right]}_{0}$

$800 - \frac{0.1(8000)}{3}$

$800 - \frac{800}{3}$

$800 - 266\frac{2}{3}$

$\underline{\underline{533\frac{1}{3}\text{ sq. units}}}$

3. Find $\int_1^3 (5t-2)^3 \; dt$.

$$= \frac{(5t-2)^4}{4(5)}\Big|_1^3 = \frac{(5t-2)^4}{20}\Big|_1^3$$

$$\left[\frac{(5(3)-2)^4}{20}\right] - \left[\frac{(5(1)-2)^4}{20}\right] = \left[\frac{(13)^4}{20}\right] - \left[\frac{(3)^4}{20}\right] = 1428.05 - 4.05$$

$$\underline{\underline{1424 \text{ sq. units}}}$$

4. Differentiate $(1+x^2)e^x$.

$$(1+x^2)\frac{d}{dx}e^x + \frac{d}{dx}(1+x^2) \bullet e^x$$

$$\underline{\underline{(1+x^2)e^x + 2x(e^x)}} \text{ or } \underline{\underline{e^x(x^2+2x+1)}} \text{ or } \underline{\underline{e^x(x+1)^2}}$$

5. Differentiate $\ln(x^3+2x+1)$.

$$\frac{1}{x^3+2x+1}(3x^2+2) = \underline{\underline{\frac{3x^2+2}{x^3+2x+1}}}$$

6. Use logarithmic differentiation to differentiate $f(x) = \frac{(x-2)^3(x-3)^4}{(x+4)^5}$. **[9 pts]**

$$\ln f(x) = \ln\left[\frac{(x-2)^3(x-3)^4}{(x+4)^5}\right]$$

$$\ln f(x) = \ln(x-2)^3 + \ln(x-3)^4 - \ln(x+4)^5$$

$$\ln f(x) = 3\ln(x-2) + 4\ln(x-3) - 5\ln(x+4)$$

$$\frac{f'(x)}{f(x)} = \frac{3}{x-2} + \frac{4}{x-3} - \frac{5}{x+4}$$

$$f'(x) = f(x)\left[\frac{3}{x-2} + \frac{4}{x-3} - \frac{5}{x+4}\right] = \underline{\underline{\frac{(x-2)^3(x-3)^4}{(x+4)^5} \bullet \left[\frac{3}{x-2} + \frac{4}{x-3} - \frac{5}{x+4}\right]}}$$

7. Determine $\int\left(\frac{2}{\sqrt{x}}-3\sqrt{x}\right)dx.$ $=\int\left(\frac{2}{x^{\frac{1}{2}}}-3x^{\frac{1}{2}}\right)dx=\int\left(2x^{-\frac{1}{2}}-3x^{\frac{1}{2}}\right)dx$

$$=\frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}-\frac{3x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{2x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$$

$$=4x^{\frac{1}{2}}-\frac{6}{3}x^{\frac{3}{2}}+C=4\sqrt{x}-2\sqrt{x^3}+C$$

8. Determine $\int\left(\frac{4}{6+t}\right)dt.$

$$=4\int\frac{1}{6+t}dt=4\ln|6+t|+C$$

9. Determine $\int\left(e^{2t}-5e^{-5t}+4\right)dt.$

$$\frac{e^{2t}}{2}-\frac{5e^{-5t}}{-5}+4t+C$$

$$\frac{1}{2}e^{2t}+e^{-5t}+4t+C$$

10. $\int(3x+2)^4\,dx=K(3x+2)^5+C.$ Find the value of K that makes the antidifferentiation formula true.

$$\frac{(3x+2)^5}{5(3)}+C=\frac{1}{15}(3x+2)^5+C$$

Therefore, $K=\frac{1}{15}$

11. A soap manufacturer estimates that its marginal cost of producing soap powder is $0.2x+1$ hundred dollars per ton at a production level of x tons per day. Fixed costs are \$200 per day. Find the cost of producing x tons of soap powder per day. [9 pts

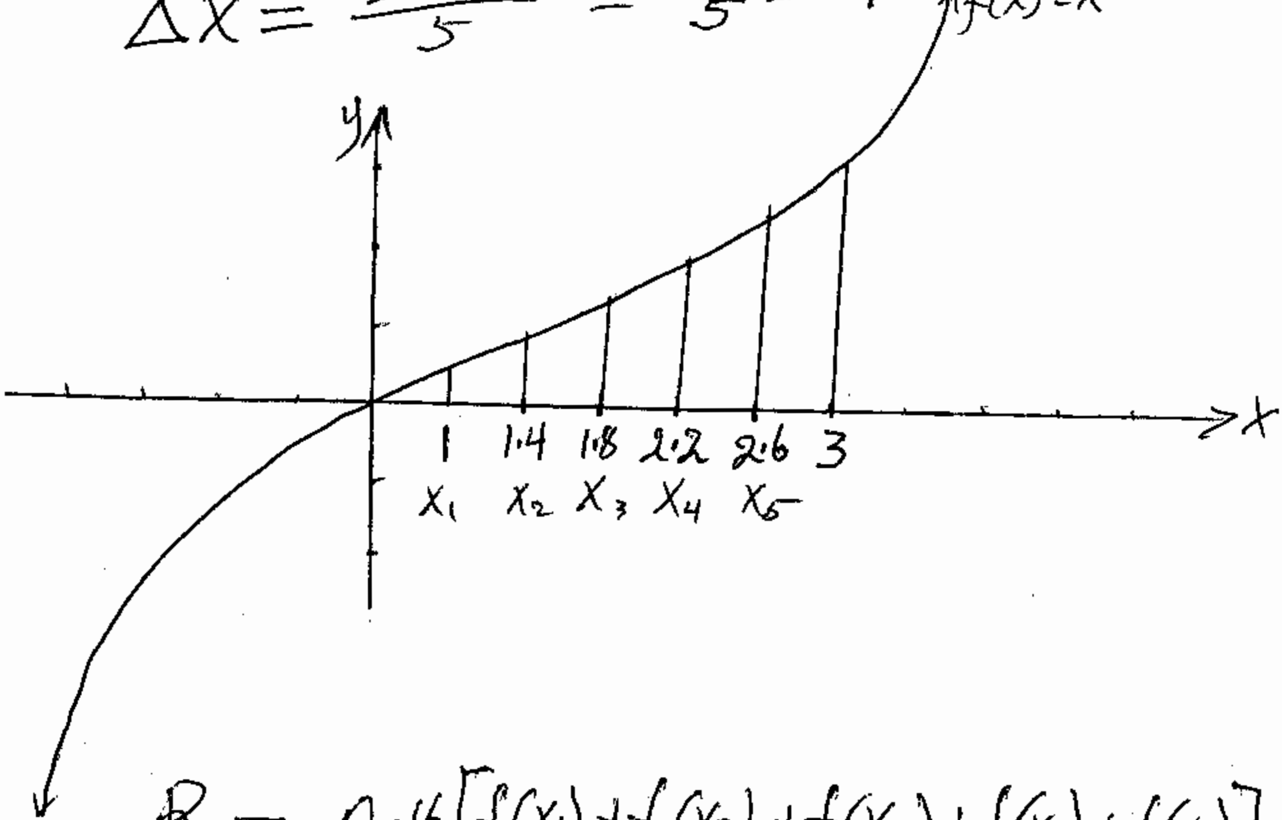
$$C'(x)=0.2x+1\Rightarrow\int C'(x)\,dx=C(x)$$

$$C(x)=\int(.2x+1)dx=\frac{0.2x^2}{2}+x+C\text{, but } C=200$$

$$\therefore C(x)=0.1x^2+x+200$$

12. Use a Riemann sum to approximate the area under the graph of $f(x) = x^3$ in the interval $1 \le x \le 3$, n = 5 using left endpoints. **[9 pts]**

$$\Delta x = \frac{3-1}{5} = \frac{2}{5} = 0.4$$



$$R = 0.4[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)]$$

$$R = 0.4[f(1)+f(1.4)+f(1.8)+f(2.2)+f(2.6)]$$

$$R = 0.4[(1)^3+(1.4)^3+(1.8)^3+(2.2)^3+(2.6)^3]$$

$$R = 0.4[1+2.744+5.832+10.648+17.576]$$

$$R = 0.4[37.8]$$

$$R = 15.12 \text{ sq. units.}$$