Core Problems.

Read the corresponding sections from the text and lecture notes. Use Maple to check your answers, or to facilitate your work.

Section 14.8: 40, 41, 43
Section 15.1: 1, 3.
Section 15.2: 5, 13 – 21 (odds only), 27, 29, 31, 35.
Section 15.3: 9, 13, 19, 21, 23, 28, 35, 37, 43, 45, 47.

Solutions of these problems should be submitted.

H9.1 Consider two lines: \( l_1 : x = 1 + t, y = 1 - t, z = 3 + 2t, t \) is any real number, and \( l_2 \), defined as intersection of two planes: \( \alpha : x + y + 2 = 0 \) and \( \beta : x - y + 2z = 4 \). Find the distance between these lines by using the following three methods.

(a) (5 points) Geometric method, which uses the projection of a vector \( \vec{AB} \) with \( A \) on \( l_1 \) and \( B \) on \( l_2 \) on the vector \( \vec{n} \) perpendicular to direction vectors of \( l_1 \) and \( l_2 \).

(b) (5 points) Write equation of the plane through \( l_1 \) which is parallel to \( l_2 \). Then find the distance from any point of \( l_2 \) to this plane by using the distance formula from a point to a plane.

(c) (5 points) By writing a parametric equation of \( l_2 \) using a parameter \( s \), and finding the absolute minimum of the function \( f(t,s) \) representing the distance (or square of the distance) between a point \( A \) on \( l_1 \) and a point \( B \) on \( l_2 \).

Make sure that the answers you get in (a), (b), (c) are equal.

H9.2 (5 points) Using Maple show that the function \( z = xe^y + ye^x \) is a solution of the equation

\[
\frac{\partial^3 z}{\partial x^3} + \frac{\partial^3 z}{\partial y^3} = x \frac{\partial^3 z}{\partial x \partial y^2} + y \frac{\partial^3 z}{\partial x^2 \partial y}
\]

H9.3 (8 points)

(i) Show that the product of the \( x \)-, \( y \)-, and \( z \)-intercepts of any tangent plane to the surface \( xyz = c^3 \) is a constant.

(ii) (2 points) Let \( \alpha \) be a tangent plane to the surface \( z = \frac{1000}{xy} \). If \( \alpha \) intersects \( x \)- and \( y \)-axes at points \( (2,0,0) \) and \( (0,5,0) \), respectively, find its intersection with \( z \)-axis.