Core Problems.

Read the corresponding sections from the text and lecture notes. Use Maple to check your answers, or to facilitate your work.

Section 13.2: 9, 15, 17, 19, 25, 29, 31, 39, 43 (can use Maple if you wish), 47.

Section 13.3: 1, 3, 5, 11-25 (odds only), 31, 40, 41, 43, 49.

Solutions of these problems should be submitted.

H5.1 (i) Find an equation of a parabola that has curvature 6 at the origin.

(ii) Find all points of the curve \( y = xe^x \) where the curvature is the greatest or the smallest, if such points exist.

Hint: (Use Maple.)

H5.2 (a) Find an equation of a line which lies on the surface \( 9x^2 - y^2 - z^2 = 0 \).

(b) Prove that there is no line lying on the surface \( 4x^3 - y^2 + 3z^4 = 1 \).

H5.3 Consider the curve \( f(t) = (2t^2 - t, e^{2t} + 3t - 2, 2e^{2t} - 6t^2 + 9t - 4) \), \( t \) is any real number.

(a) Does the curve intersect itself? Justify your answer.

(b) Is the curve planar? (i.e., is there a plane such that all points of the curve are in it). Justify your answer.

H5.4 If you present solution for part (ii), you do not have to do it for part (i).

(i) Let \( A : (0, 0, 0), B : (3, 0, 0) \). Consider the set all points \( C \) in the space such that \( AC/BC = 2 \). Prove that the set is a sphere. Find the coordinates of its center and its radius length.

Hint: Let \( C : (x, y, z) \).

(ii) Let \( A \) and \( B \) be two distinct points in space. For a fixed real number \( k \), \( 0 < k \neq 1 \), consider the set all points \( C \) in the space such that \( AC/BC = k \). Prove that the set is a sphere. Describe the position of the center and its radius length as functions of \( k \).

Hint: Introduce a Cartesian coordinate system in the space such that \( A \) becomes the origin and \( B : (1, 0, 0) \). Let \( C : (x, y, z) \).

(iii) Let \( (a, b, c) \) be the coordinates of the center of the sphere from part (ii), when the coordinate system was chosen as was suggested in the hint, and \( r \) be the length of the radius. Then \( a = a(k) \) and \( r = r(k) \).

(i) What can be said about \( \lim_{k \to 1} a(k) \), and \( \lim_{k \to 1} r(k) \)?

(ii) What can be said about \( \lim_{k \to 0^+} a(k) \), and \( \lim_{k \to 0^+} r(k) \)?