I suggest that in every assignment you start with
(i) going over the corresponding section in the text and lecture notes
(ii) solving all core problems (these you do not submit)
(iii) solving and writing solutions for the problems you have to submit.

Core Problems.

Section 12.4: 1, 3, 7, 13, 17, 19, 31, 33, 35, 39, 43, 49.
Section 12.5: read up to Planes (page 797). Do problems 1, 3, 9, 15, 19, 21.

Problems to be submitted.

H2.1 Given a cube $ABCDA_1B_1C_1D_1$, with sides $AA_1$, $BB_1$, $CC_1$ and $DD_1$ being parallel (can think of them as “vertical”).

(i) Find the angle between diagonal $AC_1$ of a cube and diagonal $AB_1$ of its face
(ii) Repeat part (i) replacing $AB_1$ by $A_1B$
(iii) Let $M$ denote the center of the square $ABCD$ and let $N$ be a point of the segment $BB_1$ such that $\frac{BN}{NB_1} = \frac{3}{2}$. Find the angle between lines $MC_1$ and $AN$.

H2.2 Let $A_1, A_2, A_3, A_4$ be four points in $\mathbb{R}^3$. Suppose lines $A_1A_2$ and $A_3A_4$ are perpendicular, and lines $A_1A_3$ and $A_2A_4$ are perpendicular. Prove that then lines $A_1A_4$ and $A_2A_3$ are also perpendicular.

H2.3 (i) If $\vec{c} = |\vec{a}|\vec{b} + |\vec{b}|\vec{a}$, where $\vec{a}, \vec{b}, \vec{c}$ are all nonzero vectors, show that $\vec{c}$ bisects the angle between $\vec{a}$ and $\vec{b}$.

(ii) Let $OA, OB, OC$ be three rays in space, and let rays $OD, OE, OF$ bisect the angles $AOB, BOC, COA$, respectively. Prove that the three angles formed by the rays $OD, OE, OF$ are either all acute, or all right, or all obtuse.

H2.4 Suppose points $A_1, A_2, \ldots, A_n$ lie on a unit circle centered at point $O$ and divide it into $n$ congruent arcs.

(i) Find
$$\sum_{i=1}^{n} |\vec{A}_iA_j|^2.$$  

(ii) Find the sum of squares of lengths of all segments $A_iA_j$, $1 \leq i < j \leq n$. 

1
(Hint: Use Problem H1.4)

H2.5 Let \( \vec{v} = 5\hat{j} \) and let \( \vec{u} \) be a vector with length 3 that initiates at the origin and rotates in the \( xy \)-plane. Find the maximum and the minimum values of the length of the vector \( \vec{u} \times \vec{v} \). In what direction does \( \vec{u} \times \vec{v} \) point?

H2.6 Prove that for any \( \vec{a},\vec{b},\vec{c} \),

\[
\vec{a} \times (\vec{b} \times \vec{c}) = (a \cdot c)\vec{b} - (a \cdot b)\vec{c}.
\]

If you wish, you can use Maple to solve this problem.