I suggest that in every assignment you start with
(i) going over the corresponding section in the text and lecture notes
(ii) solving all core problems (these you do not submit)
(iii) solving and writing solutions for the problems you have to submit.

Core Problems.

Section 12.1: 1 – 39 (only odds, excluding #37).
Section 12.2: 1 – 45 (odds only).
Section 12.3: 1 – 59 (odds only).

Problems to be submitted.

H1.1 Given two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ and point $Q$ on the segment $P_1P_2$. Suppose \( \frac{P_1Q}{P_2Q} = 5 \).

(i) Express the coordinates \((x_Q, y_Q, z_Q)\) of $Q$ in terms of the coordinates of $P_1$ and $P_2$.
(ii) Repeat part (i) replacing 5 by an arbitrary real number $k$, $k \geq 0$.

H1.2 Let $A_1, A_2, A_3, A_4$ be four points in $\mathbb{R}^3$. Let $B_{ij}$ denote the midpoint of the segment $A_iA_j$ and $C_{ijkl}$ denote the midpoint of the segment $B_{ij}B_{kl}$. Prove that points $C_{1234}$, $C_{2341}$ and $C_{1324}$ always coincide.

H1.3 Let $A_1, A_2, A_3, A_4$ be four points in $\mathbb{R}^3$. Think about them as vertices of a tetrahedron. A median of a tetrahedron is the segment which joins its vertex with the point of intersection of the three medians of the opposite face. Prove that all four medians of the tetrahedron are concurrent and their common point divides each of them in the ratio $3 : 1$.

H1.4 Suppose points $A_1, A_2, \ldots, A_n$ lie on a circle centered at point $O$ and divide it into $n$ congruent arcs. Prove that the

$$\sum_{i=1}^{n} \overrightarrow{OA_i} = \overrightarrow{0}$$

(Hint: The case when $n$ is even is easy. If $n$ is odd, consider two distinct axes of symmetry of the given set of points.)

H1.5 (i) Show that $\sum_{i=1}^{n} \cos(\alpha + \frac{2\pi}{n}i) = \sum_{i=1}^{n} \sin(\alpha + \frac{2\pi}{n}i) = 0$

(ii) Suppose that $\sin x + \sin y + \sin z = \cos x + \cos y + \cos z = 0$. Compute

$$\sin 2000x + \sin 2000y + \sin 2000z$$

(Hint: in both (i) and (ii) one can use the result from H1.4.)