
Present your solutions with all details. All problems carry the same weight of 10 points; 90 points total. Do not copy the problems. You have 1:30 minutes for this part of the exam.

1. Find the distance from the point \( P(1, 1, 1) \) to the line through \( Q(0, 6, 8) \) and \( R(-1, 4, 7) \). Explain your solution.

2. Let \( |\vec{a}| = 2, |\vec{b}| = 3 \), and the measure of the angle between \( \vec{a} \) and \( \vec{b} \) is \( \pi/3 \). Find the cosine of the angle \( \theta \) between \( \vec{b} - \vec{a} \) and \( 2\vec{a} - \vec{b} \). Explain your solution.

3. Prove that a trajectory of a planet is a plane curve (i.e., all points of the curve are in a plane).

4. Evaluate the line integral \( \oint_C xd\mathbf{i} + yd\mathbf{j} \), where \( C \) consists of the line segment from \((0,1)\) to \((0,0)\) and from \((0,0)\) to \((1,0)\) and the parabola \( y = 1 - x^2 \) from \((1,0)\) to \((0,1)\), by two different methods:
   (a) directly
   (b) by using Green’s Theorem.

5. If \( f(x, y) = xy^4/(x^2 + y^8) \), does \( \lim_{(x,y)\to(0,0)} f \) exist? Explain your answer.

6. A rectangular box without a lid is to be made from 12 \( \text{m}^2 \) of cardboard. Find the maximum volume of such a box. Explain your solution.

7. Find an equation of a parabola that has curvature 4 at the origin.

8. Given a ball of radius \( R > 0 \) of uniform density \( \delta \), and a point mass \( m \) of distance \( a > R \) from the center of the ball. Assume the gravitational constant be \( G \).
   (i) Reduce the computation of the force to the iterated triple integral, and explain how the bounds of integration are obtained. You do NOT have to demonstrate the computation of the integral.
   (ii) Write down the result of the computation of the triple integral in part (i).

9. Determine whether or not the vector field
   \[
   \vec{F}(x, y, z) = 2xy \mathbf{i} + (x^2 + 2yz) \mathbf{j} + y^2 \mathbf{k}
   \]
   is conservative. If it is conservative, find a function \( f \) such that \( \vec{F} = \nabla f \).
Please solve these problems using Maple. Print your name at the beginning of the worksheet. Delete from the worksheet all unnecessary lines, print it and submit your work. All problems carry the same weight of 10 points; 40 points total. Do not copy the problems. You have 30 minutes for this part of the exam.

**Problem 1.** Find all real values of \( k \) for which the volume of the parallelepiped determined by the vectors \( \overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD} \), where \( A(1, 2, 10), B(20, -13, 1), C(-1, 2k, 1), D(3k-1, k, -2) \) is equal to 1000. Find an approximate value of the result if needed.

**Problem 2.** Let \( E \) be the solid in the first octant bounded by the cylinder \( x^2 + y^2 = 1 \) and planes \( y = z, x = 0 \) and \( z = 0 \) with density function \( \rho(x, y, z) = 1 + x + y + z \). Find the coordinates of the center of mass of \( E \).

**Problem 3.** Show that the curve \( \overrightarrow{r}(t) = \langle t, t^2, t^3 \rangle, t \geq 0 \), is not planar.

**Problem 4.** (i) Find the area of the surface with vector equation

\[
\overrightarrow{r}(u, v) = \langle \cos^3 u \cos^3 v, \sin^3 u \cos^3 v, \sin^3 v \rangle,
\]

where \( 0 \leq u \leq \pi, \quad 0 \leq v \leq 2\pi \).

(ii) approximate your answer in (i).