
Present your solutions with all details. All problems carry the same weight of 10 points; 60 points total. Do not copy the problems. You have 50 minutes for this part of the exam.

1. Write down carefully the following definitions:
   (i) of the double integral of \( f(x,y) \) over the rectangle \( R \);
   (ii) of the moment of a lamina with mass density function \( \rho(x,y) \) with respect to \( x \)-axis;
   (iii) of the the 3-dimensional vector field;
   (iv) of the polar rectangle.

2. Use Lagrange multipliers to find the minimum and the maximum values of the function \( f(x,y,z) = 2x + 6y + 10z \) on \( x^2 + y^2 + z^2 = 35 \).

3. Explain the formula for the volume \( dV \) in spherical coordinates as \( \rho^2 \sin \phi \, d\rho \, d\theta \, d\phi \).

4. Evaluate the integral by first reversing the order of integration:
   \[
   \int_0^1 \int_{3w}^3 e^{x^2} \, dx \, dy.
   \]

5. (i) Consider the region \( D \) in the 1-st quadrant bounded by the rose \( r = \cos 3\theta \) and the \( x \)-axis.
   (a) Sketch the region.
   (b) Rewrite the integral \( \iint_D \, dA \) representing the area of \( D \) as an iterated integral indicating clearly the limits of integration. You do not have to compute the integral.
   (ii) Represent the work done by the force field \( \vec{F} = x\vec{i} + (y + 2)\vec{j} \) in moving the object along the arch of the cycloid
   \[
   \vec{r}(t) = (t - \sin t)\vec{i} + (1 - \cos t)\vec{j}, \quad 0 \leq t \leq 2\pi
   \]
   by an integral of a simple expression \( f(t) \) with respect to \( t \). You do not have to compute the integral.

6. Show that the magnitude \( F \) of the force of gravitational attraction of a lamina with constant density \( \rho \) that occupies an entire plane on a point-mass object with mass \( m \) located at a distance \( d \) from the plane is \( F = 2\pi Gm\rho \).
Problem 1. Let $E$ be the solid in the first octant bounded by the cylinder $x^2 + y^2 = 1$ and planes $y = z$, $x = 0$ and $z = 0$ with density function $\rho(x, y, z) = 1 + x + y + z$. Find

(i) the exact value of the mass of $E$

(ii) approximate the result in (i).

Problem 2. (i) Find the exact value of

$$\int_C x^3 y^2 z \, ds,$$

where $C$ is the curve with parametric equations $x = e^{-t} \cos 4t, y = e^{-t} \sin 4t, z = e^{-t}, 0 \leq t \leq 2\pi$.

(ii) approximate the result in (i).