
Present your solutions with all details. All problems carry the same weight of 10 points; 60 points total. Do not copy the problems. You have 45 minutes for this part of the exam.

1. Write down carefully the following definitions:

   (i) the curvature of a curve at a given point;

   (ii) normal plane of a curve at a point;

   (iii) linear approximation (or tangent plane approximation) of a function of two variables $z = f(x, y)$ at a point $(x_0, y_0)$;

   (iv) the directional derivative of a function $z = f(x, y)$ at a point $(x_0, y_0)$ in the direction given by a unit vector $\mathbf{u} = \langle a, b \rangle$.

2. (i) Does the curve $\mathbf{r}(t) = \langle \sin t, e^{t^2}/t, t^3 + 5t \rangle$, $t > 0$, intersect itself? Justify your answer.

   (ii) Find an equation of the tangent plane to the elliptic paraboloid $z = 2x^2 + y^2$ at point $(1, 1, 3)$.

3. (i) Find an equation for the surface obtained by rotating of a parabola $y = x^2$ about $y$-axis.

   (ii) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \langle 2t, 3t^2, \sqrt{t} \rangle$ and $\mathbf{r}(1) = (1, 1, 0)$.

4. Prove that if $f$ is a differentiable function of $x$ and $y$, then $f$ has a directional derivative in the direction of any unit vector $\mathbf{u} = \langle a, b \rangle$ and

$$D_{\mathbf{u}} f(x, y) = f_x(x, y)a + f_y(x, y)b.$$  

5. If

$$f(x, y) = \frac{xy^2}{x^2 + y^4},$$

does $\lim_{(x, y) \to (0, 0)} f(x, y)$ exist? Explain your answer.

6. A rectangular box stands on horizontal floor. A ball rolls off a top of the box with a (horizontal) speed of 4 ft/s directed perpendicular to the edge. The box is 5 ft high. How far from the vertical wall of the box will the ball fall on the floor? You do not have to approximate your answer.
Please solve these problems using Maple. Print your name at the beginning of the worksheet. Delete from the worksheet all unnecessary lines, print it and submit your work. All problems carry the same weight of 10 points; 20 points total. Do not copy the problems. You have 15 minutes for this part of the exam.

**Problem 1.** Find and classify the critical points of the function

\[ f(x, y) = 10x^2y - 5x^2 - 4y^2 - x^4 - 2y^4. \]

Find an approximate value of local extrema (if they exist).

**Problem 2.** Demonstrate that for every two differentiable vector functions \( \vec{u} = \vec{u}(t) \) and \( \vec{v} = \vec{v}(t) \),

\[ \frac{d}{dt} [\vec{u}(t) \times \vec{v}(t)] = \vec{u}'(t) \times \vec{v}(t) + \vec{u}(t) \times \vec{v}'(t). \]