M349-010 (Prof. Hsiao) Additional Hourly Exam III due May 18, 2001

NOTE: SHOW ALL YOUR WORK. ALL PROBLEMS ARE EQUALLY WEIGHTED. (*You may use MATLAB to perform your computations, whenever it is convenient to you.*)

1. Given the matrix

$$A = \begin{bmatrix} 1 & -2 & 1 & 1 & 2 \\ -1 & 3 & 0 & 2 & -2 \\ 0 & 1 & 1 & 3 & 4 \\ 1 & 2 & 5 & 13 & 5 \end{bmatrix},$$

   find the following:
   (a) the rank of A, (b) a basis for the null space of A, (c) a basis for the row space of A and (d) a basis for the column space of A.

2. Let v be an inner product space. Suppose that W be a subspace of V and $w \neq 0$ be a fixed vector in W. Show that for any $v \in V$,

$$\alpha := \frac{(v, w)}{||w||^2}$$

   is the unique scalar such that

$$w^{\perp} := v - \alpha w$$

   is orthogonal to w.

3. Let $C[-\pi, \pi]$ be the inner product space of real-valued continuous functions on the interval $[-\pi, \pi]$ with inner product defined by

$$(f, g) := \int_{-\pi}^{\pi} f(t)g(t)dt, \quad f, g \in C[-\pi, \pi].$$

   (a) Use the Gram-Schmidt process to obtain an orthonormal basis for the subspace $W = span\{1, cos\ t, sin\ t\}$ of the inner product space.

4. Let W be the subspace of continuous functions on $[-\pi, \pi]$ in Problem 3. Write the vector $f = t - 1$ as

$$f = w + w^{\perp}$$

   with $w \in W$ and $w^{\perp} \in W^{\perp}$.

5. Let W be the plane : $3x + 2y - z = 0 \in \mathbb{R}^3$.
   (a) Show that

$$W^{\perp} := \{v \in \mathbb{R}^3 \mid v \perp W\}$$

   is a subspace of $\mathbb{R}^3$.
   (b) Find a basis for the subspace $W^{\perp}$.