
Math 302-010, ASSIGNMENT 2

Due: Thursday, July 10

Directions: Write clearly and neatly. Explain yourself fully, and show all work. The quality of your argument is worth more than the correctness of you answer!

1. (3 pts.) Find a solution to the differential equation

$$y'' + xy = 0$$

by building a power series for y about the point $x = 0$. As with similar examples we did in class, your solution should be in terms of two unknown coefficients.

2. (4 pts.) Several weeks ago we considered a mass-spring system. We derived the differential equation

$$my'' + \gamma y' + ky = 0$$

to model the position y of the mass as a function of time, where m is the object's mass, γ is the damping coefficient, and k is the spring constant. Our goal here is to turn this equation into a system of first order equations and consider the stability of the resulting equilibrium solution.

- Write this second order linear equation as a 2×2 system of first order linear equations by setting $x = y'$. Put the system into matrix form.
 - Use the fact that the constants m , γ , and k are all nonnegative to show the coefficient matrix has a nonzero determinant. This proves the only equilibrium solution is $x = y = 0$. What is the physical significance of this equilibrium solution (i.e. how is the mass-spring system behaving at this equilibrium solution)?
 - Find the eigenvalues of the coefficient matrix, and consequently classify the equilibrium solution (source, sink, saddlepoint, spiral, or center) in the case where the damping coefficient γ is zero. Is this answer consistent with the physical behavior we would observe in this case? Why or why not?
 - Classify the equilibrium solution in the case where γ is greater than zero. Again, is this answer consistent with the physical behavior we would observe in this case?
3. (4 pts.) Consider the system

$$x' = 3x + y, \quad y' = -6x - 2y$$

where both x and y are functions of the variable t . Although this system is linear, its solutions behave differently than in other examples we have seen. The reason is that the coefficient matrix has a determinant of zero, so there are infinitely many equilibrium solutions.

- a. All of the equilibrium solutions live on the same line. What is the equation of that line?
 - b. Find the eigenvalues and corresponding eigenvectors of this system, and hence find the general solution to the system.
 - c. Use Maple to plot several trajectories for solutions in the phase plane. Describe the behavior of solutions as $t \rightarrow \infty$.
4. (4 pts.) For each of the following systems, find all equilibrium solutions (do this by hand). Then use Maple to plot the direction field and several trajectories in the phase plane. Use these trajectories to classify the equilibrium solutions.
- a. $x' = x - xy, \quad y' = y + 2xy.$
 - b. $x' = (2 + x)(y - x), \quad y' = (4 - x)(y + x).$
 - c. $x' = 4 - 2y, \quad y' = 12 - 3x^2.$