Math 302-010, ASSIGNMENT 1

Due: Wednesday, June 25

Directions: Write clearly and neatly. Explain yourself fully, and show all work. The quality of your argument is worth more than the correctness of you answer!

2.1.21:(2 points) For part (a), use Maple to draw the direction field. Answer the questions posed in part (a) based solely on the shape of the direction field (no calculations). Print out a copy of the direction field and include it with your submission. Do part (b) by hand. You'll probably need to use integration by parts twice (it's kind of a trick).

2.1.33:(2 points) First suppose $a = \lambda$ and solve the equation via the standard techniques (method of integrating factors). To verify that $y \to \infty$ as $t \to \infty$ you might want to recall L'Hopital's Rule. Then solve the equation again, but this time under the assumption $a \neq \lambda$. In this second case, verifying $y \to \infty$ as $t \to \infty$ is more straightforward.

2.2.24:(2 points) Find the solution by hand, and then graph the solution with Maple. Find the interval over which the solution exists, accurate to three decimal places (this can be done in Maple). Include a copy of the graph and accompanying calculations with your submission. The back of the book tells you the x-value at which the solution attains its maximum, so it's no secret. Using the ideas you learned for finding the maximum and minimum values of functions from calculus, provide a compelling reason why the solution attains its maximum value at this point.

2.5.26:(2 points) All the work in this problem can be done by hand, although you're free to use Maple or a graphing calculator to help you envision the cubic curves $ay - y^3$ you're considering. This problem will only make sense if you've already read through problem 25 and the preceeding discussion on bifurcation points.

3.2.27:(2 points) This one is fairly straightforward, so no additional comments are needed here.

3.4.28:(2 points) This is an interesting exercise that provides an alternate way to derive Euler's Formula without invoking power series. Instead, it shows how the complex exponential function e^{it} arises as a solution to a familiar differential equation. By using the theory we've learned about fundamental sets of solutions to linear homogeneous equations, Euler's Formula can be deduced.

3.5.20:(3 points) In solving the equation $ay''+by'+cy = 0$, we considered the case where the characteristic equation has one repeated (real) root. In this case the characteristic equation only gives us one particular solution, so to complete a fundamental set of solutions we had to employ a clever trick to find a second linearly independent solution. If you found that trick unsatisfactory (I do), then this problem outlines a more mathematically-sound method for arriving at the second solution.

Part (a) is basically observing that any equation $ay'' + by' + cy = 0$ whose characteristic has repeated roots must have a very particular shape. Part (b) is straightforward. The real work happens in part (c), where you basically have to reverse-engineer the Wronskian $W(y_1, y_2)$. Since you know y_1, and since you know the value of $W(y_1, y_2)$, this lets you set up a *first order* differential equation for y_2. Upon solving this differential equation, you will have your second solution y_2. *Note:* Since y_2 is meant to be a particular solution, you are free to assign whatever values you like to any constants appearing in your solution for y_2.