Robert Pertsch-Rüsterholz Gilbert
Department of Mathematics
I wasn’t always a mathematician

Originally I was a student of physics and obtained a BSc and MSc in Physics before switching to mathematics. The cause was the *Heisenberg Uncertainty Principle*, which led me to write to Albert Einstein about my uncertainty about *Uncertainty*. After switching to the Mathematics Department I worked on a project with Hirsh Cohen and it resulted in a paper. Hirsh left Carnegie for Rennselaer. I began working with Zeev Nehari, who had recently joined the Department from Washington University. This was professionally a good move, as Zeev Nehari was one of the preeminent complex analysts during the 1950s.
THE INSTITUTE FOR ADVANCED STUDY
PRINCETON, NEW JERSEY

March 19, 1963

SCHOOL OF MATHEMATICS

Mr. Robert F. Gilbert
Physics Dept.
Carnegie Institute of Technology
Pittsburgh, Pa.

Dear Mr. Gilbert:

I am sending you under separate cover a very short paper about the question you are interested in. The essential thing is that one cannot consider as true the following two statements together:

1) The $\gamma$-function is giving a complete description of an individual case.

2) There is no immediate coupling of things spatially separated.

Sincerely yours,

[Signature]

Albert Einstein.
This is the paper with Hirsh Cohen on free-boundary problems, an area I would return to after many years.
For my PhD thesis I used *Integral Operator Methods*, á le Stefan Bergman, which were used to determine the analytic and singular behavior of various classes of elliptic partial differential equations. The Bergman-Whittaker Operator generates harmonic functions in \( \mathbb{R}^3 \),

\[
H(X) = \mathcal{B}_3(f) := \int_C f(u, \zeta) \frac{d\zeta}{\zeta},
\]

from functions of two complex variable \( f(u, \zeta) \), where \( X = (x_1, x_2, x_3) \), \( u = x_1 (\zeta - 1/\zeta) + ix_2 (\zeta + 1/\zeta) + x_3 \) is an auxiliary variable and \( C \) is a contour in \( \mathbb{C}^1 \). I proved necessary and sufficient conditions for \( H(X) \) to be singular in terms of its analytic *associate*. 
Theorem

Let the defining function for the set of singularities of \( f(u, \zeta)\zeta^{-1} \) be a global function in \( \mathbb{C}^2 \). Then if \( h(u, \zeta) := S(X; \zeta) = 0 \) is such a function we have that \( H(X) = B_3f \) is regular for all points \( X \) which may be reached by continuation along a curve \( \Gamma \) starting at an initial point of definition \( X_0 \), provided \( X \) and hence, the curve \( \Gamma \), do not lie on the intersection

\[
S := \{X : S(X; \zeta) = 0\} \cap \{X : S_\zeta(X; \zeta) = 0\}.
\]
I wasn't always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor

Indiana University Bloomington, Professor 1966-75

Analytic Methods in Mathematical Physics

My First Monograph

Generalized Hyperanalytic Function Theory

The Georgians

Free University Berlin and the Hahn-Meitner Institute 1974-75

University of Delaware, Unidel Professor 1975

Some Work in Elasticity, etc.

I visit China for the First Time, 1984

Research in Underwater Acoustics

The 65th Birthday Conference in Graz

The Cold War Ends: No More Underwater Acoustics

The Birth of ISAAC

My 70th Birthday Conference in Frejus, France

The 75th Birthday Conference in Orlando

Medical Mathematics

Figure: Zeev Nehari, né Wili Weissbach
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Figure: Graduation, Carnegie-Mellon 1958
Using the methods of projective differential geometry we were able to prove, among other results,

**Theorem**

Let $H(X)$ be a harmonic function given by $H(X) = B_3 f$ where the singularities of $f(u, \zeta)$ have a global representation, $h(u, \zeta) := \zeta u - \phi(\zeta) = 0$. Then the singularities of $H(X)$ must lie on a develople in $\mathbb{P}$. Further, if $\phi(\zeta)$ is a polynomial of degree $n, n > 2$, then the singularities are twisted $n^{th}$ degree (complex) space curves.
The above results were generalized to harmonic functions in $\mathbb{R}^4$ using the operator

$$H(\mathbf{X}) = \mathbf{G}_4 f := -\frac{1}{4\pi^2} \int_{C_1} \frac{d\xi}{\xi} \int_{C_2} \frac{d\eta}{\eta} f(\tau, \xi, \eta) \quad \mathbf{X}; = (x_1, x_2, x_3, x_4)$$

where $f$ is a holomorphic function of three complex variables and

$$\tau := x_1 \left(1 + \frac{1}{\eta \xi}\right) + ix_2 \left(1 - \frac{1}{\eta \xi}\right) + x_3 \left(\frac{1}{\xi} - \frac{1}{\eta}\right) + ix_4 \left(\frac{1}{\xi} + \frac{1}{\eta}\right).$$

Here the curves $C_k$ are homologous to zero. Kreyzig inverted this operator and referred to $G_4$ as Gilbert’s operator. Using Kreyzig’s inversion the entire three-dimensional theory was carried over to either $\mathbb{R}^4$ or $\mathbb{C}^4$. 
A Personal Mathematical History

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The 75th Birthday Conference in Orlando

Medical Mathematics

Figure: Erwin Kreyzig
Institute for Fluid Dynamics and Applied Mathematics
1961-65 Assistant Professor and Associate Professor

We were supported by Alexander Weinstein’s AFOSR grant and our NSF grant to make a systematic investigation of the analytic properties of solutions to singular, elliptic partial differential equation of the type blow

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{2\mu}{y} + \frac{\partial u}{\partial y} = 0
\]

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{2\mu}{y} + \frac{\partial u}{\partial x} + \frac{2\nu \partial u}{y \partial y} = 0
\]

\[
\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial y^2} + \frac{2\mu}{y} + \frac{\partial u}{\partial x} + \frac{2\nu \partial u}{y \partial y} + k^2 u = 0, \quad \text{etc.,}
\]

in two and higher dimensions.
We developed the so-called \textit{envelope method} which was applied to potential scattering and to quantum field theory by Gilbert with Aks, Howard and Shieh.
A Personal Mathematical History

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The 75th Birthday Conference in Orlando

Medical Mathematics

Figure: Expansions of analytic functions
Indiana University Bloomington, Professor 1966-75

I am all of 34 and Full Professor at Indiana University. I have an AFOSR grant which is supporting, in addition to me, David Colton, Lucio Tavernini and Kendall Atkinson. I start the journal *Applicable Analysis*.

*Figure:* Kendal Atkinson and Lothar Collatz. During the 1960s Collatz was the premier numerical analyst in Germany.
Figure: Sketch of "Owls" by L. Collatz. Collatz knew I liked owls and gave me the sketch.
A Personal Mathematical History

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Medical Mathematics

Figure: Max Schiffer visits us at IU.
Analytic Methods in Mathematical Physics

In June 1968 Roger Newton and I organized an AFOSR Conference named *Analytic Methods in Mathematical Physics*
In his plenary address Maria v. Krzywoblocki presented a review of the integral Operator Method as initiated by Bergman and others, me included.
A Personal Mathematical History

I wasn't always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor

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Analytic Methods in Mathematical Physics

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Medical Mathematics

Figure: Paul Garabedian and David Widder
A Personal Mathematical History

I wasn't always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor

Indiana University Bloomington, Professor 1966-75

Analytic Methods in Mathematical Physics

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Medical Mathematics

Figure: A. Wheeler and S. Abyankar
A Personal Mathematical History

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Medical Mathematics

Figure: Bers, Garabedian, Erdelyi, Masani, me
My First Monograph

I summed up my work to this time with the Academic Press monograph
Before leaving the operator approach of Bergman I developed the *Method of Ascent MoA*. The method of ascent maps a harmonic function of two variables onto solutions of a *metaharmonic*-type equation.

\[
\frac{\partial^2 u}{\partial x_1^2} + \ldots + \frac{\partial^2 u}{\partial x_n^2} + F(r^2)u = 0, \quad r^2 = \|x\|
\]

This idea has had applications in potential scattering and acoustical scattering. Robert Carroll referred to this as the Bergman-Gilbert *Transmutation*. 
SOME REMARKS
ON THE BERGMAN-GILBERT
INTEGRAL OPERATOR

By Robert CARROLL

1. Introduction

There is a considerable literature on integral operators which transform analytic functions (or harmonic functions) into solutions of elliptic equations. Historically the principal impetus seems to have been Bergman's extensive work on the subject and subsequently important contributions were made by numerous authors; we cite here only the summary treatments ([3], [12], [13], [14], [21], [22], [24], [30]). In particular certain (direct and inverse) problems in scattering theory have been investigated using such operators (c.f. [12], [15], [16], [17], [18], [19], [21]). The author and F. Santosa in [9] have been more or less systematically treating certain singular inverse problems via transmutation methods (see e.g. [4] for transmutation and c.f. also [6], [7]; typical problems here involve geophysical situations with some kind of spherical or cylindrical symmetry as in [11], [23], [28]). In the process of comparing and unifying various methods and points of view we were led to look for a complete transmutational formulation for what we shall call the Bergman-Gilbert operator. In view of the importance of this Bergman-Gilbert operator the author's solution is presented here as a separate note.

Thus in particular we show how the Bergman-Gilbert operator can be characterized as a certain transmutation \( B : P^* \rightarrow Q^* \) (c.f. (2.2) and (2.11) for \( P^* \) and \( Q^* \)) whose kernel can be represented by a "spectral" pairing of suitable eigenfunctions of \( P^* \) and \( Q^* \) (Theorem 5.3). This places the Bergman-Gilbert operator in the context of a general transmutation theory for operators of the form \( Q^* \) in (2.11) and in particular this allows one to use known information about the Bergman-Gilbert operator to produce transmutations and connection formulas between special functions. Such a transmutation theory is important in dealing with transmutations of Laplace operators Lemma 2.5 (and the associated scattering problems in [12], [13], [15], [17], [18], [19], [21], for example) as well as in treating scattering problems at fixed energy in quantum mechanics (c.f. [8], [10]). The spectral variables which
I then turned away from the Bergman approach to function theoretic methods for pde to that of Bers and Vekua. and to the introduction of an entirely new function theory called *Generalized Hyperanalytic Function Theory* \textbf{GHA}.
The generalized hyperanalytic theory was developed to handle individual blocks of first order elliptic systems, ie those terms belonging to a single complex eigenvalue. The GHA functions satisfied equations of the type

\[ \mathbf{D} u + A u + B \bar{u} = F, \quad \mathbf{D} := \frac{\partial}{\partial \bar{z}} + q(z) \frac{\partial}{\partial z} \]

with \( q(z) := \sum_{k=1}^{n} e^{k} q_{k}(z), \quad A = \sum_{k=0}^{n} e^{k} a_{k}(z) \) etc. where \( e \) is a nilpotent such that \( e^{n} = 0 \). A complete function theory was developed for this system by Gilbert with Hile, Begehr, Buchanan, Wendland etc. Bojarski also treated this problem using a different approach. Gillbert with Buchanan and Wendland used this function theory to investigate elastic plates and shells.
GENERALIZED HYPERCOMPLEX FUNCTION THEORY(1)

BY

ROBERT P. GILBERT AND GERALD RILEY

ABSTRACT. Lipman Bers and Ilya Vekua extended the concept of an analytic function by considering the distributional solutions of elliptic systems of two equations with two unknowns and two independent variables. These solutions have come to be known as generalized (or pseudo) analytic functions. Subsequently, A. Douglis introduced an algebra and a class of functions which satisfy (classically) the principal part of an elliptic system of 2r equations with 2r unknowns and two independent variables. In Douglis’ algebra these systems of equations can be represented by a single “hypercomplex” equation. Solutions of such equations are termed hyperanalytic functions. In this work, the class of functions studied by Douglis is extended in a distributional sense much in the same way as Bers and Vekua extended the analytic functions. We refer to this extended class of functions as the class of generalized hyperanalytic functions.

1. The equation and the algebra. A. Douglis [1] showed that an elliptic system of the first order in two independent variables, with sufficient smoothness requirements on the coefficients of the first order terms, can be decomposed into the canonical subsystems

\[
\begin{align*}
\mu_{0,x} - \nu_{0,y} + \cdots &= 0, \\
\mu_{0,y} + \nu_{0,x} + \cdots &= b_0, \\
\mu_{k,x} - \nu_{k,y} + au_{k-1,x} + bu_{k-1,y} + \cdots &= 0, \\
\mu_{k,y} + \nu_{k,x} + au_{k-1,x} + bu_{k-1,y} + \cdots &= b_k,
\end{align*}
\]

where the dots represent zero order terms. He called the system a generalized Beltrami system if it is homogeneous and contains no terms of zero order. He showed that with a certain commutative algebra such a system can be written in a

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Key words and phrases. Hypercomplex variables, pseudo analytic functions of Bers and Vekua, elliptic systems.

(1) This research was supported in part by the Air Force Office of Scientific Research through AF-AFOSR Grants No. 71-2205A, and No. 74-25992.
The Georgians

Ilya Vekua invites me to attend the Muskhelishvili Conference in Tblisi. I meet Ladyshenskya, Mazya and Koshelev and Leray. I already knew Fichera, Vekua and Oleinik from Maryland.

Figure: Gaetano Fichera with Galina Koshelev
A Personal Mathematical History

I wasn’t always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor

Indiana University Bloomington, Professor 1966-75

Analytic Methods in Mathematical Physics

My First Monograph

Generalized Hyperanalytic Function Theory

The Georgians

Free University Berlin and the Hahn-Meitner Institute 1974-75

Some Work in Elasticity, etc.

I visit China for the First Time, 1984

Research in Underwater Acoustics

The 65th Birthday Conference in Graz

The Cold War Ends: No More Underwater Acoustics

The Birth of ISAAC

University of Delaware, Unidel Professor 1975

I am 63 and I begin to work in Homogenization

My 70th Birthday Conference in Frejus, France

The 75th Birthday Conference in Orlando

Medical Mathematics

Figure: Vladimir Mazya and Koshelev
Free University Berlin and the Hahn-Meitner Institute 1974-75

I spent a sabbatical semester Spring 1974 at the Hahn Meitner Institute for Nuclear Research in Berlin. The Mathematics group was headed up by Wolfgang Haack who happens to be Wolfgang Wendland’s thesis advisor. The possibility to take over the Dinghas Chair at the Free University opened up and I remained in Berlin for another year. This time was spent developing the generalized hyperanalytic function theory with Heinrich Begehr and Manfred Schneider. George Hsiao and I carry on a midnight vigil writing papers, he in Darmstadt and me in Berlin. I visit the Soviet Union and the Georgian Republic again.
I wasn’t always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor
Indiana University Bloomington, Professor 1966-75

Analytic Methods in Mathematical Physics
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Generalized Hyperanalytic Function Theory
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Medical Mathematics

Figure: A paper of mine and George’s and Pat’s in *Rendiconti*.”
I visit the Humboldt University and Martin Luther University and begin to lecture in German.

Figure: Christian Vidic, Hahn-Meitner Institute

Figure: Wolfgang Tutschke and Naas at Humboldt University
I wasn’t always a mathematician

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Medical Mathematics

Figure: My father who is my present age

Figure: My father who is my present age
University of Delaware, Unidel Professor 1975

I am 43 and Unidel Professor. The AFOSR allows me to transfer the grant to Delaware. The grant supports my work on generalized hyperanalytic functions and elasticity and David Colton’s work on scattering. Alan Jeffrey and I had a DOE grant here also for a while. I become interested in Underwater Acoustics after spending a summer the Naval Underwater Systems Center in New London, CT. I had served on David Wood’s PhD committee at the University of Rhode Island. David had done work on the Function Theoretic Method for his thesis. This leads to support from NOAH and NSF for this research. David Wood and I wrote a paper on the use of transmutations in underwater acoustics and provided some propagator representations. These were representations in a stratified, acoustic wave guide. This work was continued by me with Steve Xu and published jointly in what now is a well cited set of articles.
Some people at and passing through Delaware during my first few years here.
I wasn’t always a mathematician

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Medical Mathematics

Figure: Pat Brown, David Colton, Alan Jeffrey
A Personal Mathematical History

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Figure: Tom Angell

Figure: George Hsiao
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Medical Mathematics

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Function theoretic solutions to problems of orthotropic elasticity *

ROBERT P. GILBERT 1,** and LIN WEI 2

1 Department of Mathematics, University of Delaware, Newark, DE 19711, USA
2 Zhongshan University, Guangzhou, People's Republic of China

(Received October 18, 1983)

Abstract

The plane strain problem for a two dimensional orthotropic elastic body is investigated. In particular analytic representations for the solution of the displacement boundary value problem and the stress boundary value problems are found. To this end, the Navier equations are reduced by means of composite transformations to normal form. These are the so-called equations for bianalytic function of the type (hk). The generalized Cauchy integral formula for this function theory is used to obtain representation formulae. A simplified method to solve these problems by bianalytic function theory is given for certain situations of plane strain for an orthotropic elastic body. AMS (MOS): 35A20, 35C05, 35G15, 35J55.

1. Introduction

Function theoretic methods for second order elliptic systems have been investigated by Hua Loo Keng [1], [2] and his coworkers. These results are well known in the east; however, for the benefit of western readers we shall present as preliminaries a short introduction to these ideas.

Let us consider the following second order system

\[ A \frac{\partial^2 u}{\partial x^2} + 2B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} = 0, \]

where \( A, B, C \) are constant \( 2 \times 2 \) real matrices. We shall refer to this system as elliptic if the characteristic equation \( \det(IA - 2B + C) = 0 \) possesses only complex eigenvalues. When this characteristic equation has two multiple (complex) eigenvalues the system (1.1) may be reduced to the canonical form (1.2):

\[ \begin{bmatrix} 1 & 0 \\ -\lambda & 1 \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \frac{\partial}{\partial y} u = 0, \]

(1.2)

for \( \lambda \neq 0, -1 \). This factors into the first-order system

\[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \frac{\partial}{\partial y} u = 0, \]

(1.3)

* The work of this author was supported in part by grant no. DE-AC01-81ER-10967 from the Department of Energy.
A Personal Mathematical History  I wasn't always a mathematician  Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor  Indiana University Bloomington, Professor 1966-75  Analytic Methods in Mathematical Physics  My First Monograph  Generalized Hyperanalytic Function Theory  The Georgians  Free University Berlin and the Hahn-Meitner Institute 1974-75  University of Delaware, Unidel Professor 1975  Some Work in Elasticity, etc.  I visit China for the First Time, 1984  Research in Underwater Acoustics  The 65th Birthday Conference in Graz  The Cold War Ends: No More Underwater Acoustics  The Birth of ISAAC  I am 63 and I begin to work in Homogenization  My 70th Birthday Conference in Frejus, France  The 75th Birthday Conference in Orlando  Medical Mathematics

The Two-Dimensional, Linear, Orthotropic Plate: The Traction Problem

R. P. Gilbert, Newark, and M. Schneider, Karlsruhe

Introduction

In [13] we extended the analysis of Ciarlet and Destuynder [5] (see also [1], [4], and [7]) to the clamped orthotropic plate. For the present paper we shall apply these methods to the orthotropic plate under traction. In particular, we shall be considering the type of problem posed in Friedrichs and Dressler [10] for the isotropic plate and make use of the fact that the variational problem will split just as was the case for the partial differential equation formulation. With the present approach we shall be able to produce a formal asymptotic analysis for the formal asymptotics used in Friedrichs and Dressler.

So that present results may be compared with [5] and [13] we use the same notation. Partial derivatives are indicated by

\[ \frac{\partial v}{\partial x_i}, \quad \frac{\partial^2 v}{\partial x_i \partial x_j}, \]

where Greek indices take the values \(1, 2\), whereas Latin indices take the values in \(1, 2, 3\). Repeated indices mean the summation convention is being followed. Following [5] we introduce the norms

\[ \| \mathbf{v} \|_{1/2} := \left( \int_{\Omega} |\mathbf{v}|^2 \, dx \right)^{1/2}, \]

\[ \| v \|_{\Omega, \Gamma} := \left( \int_{\Gamma} \left( \frac{\partial v}{\partial n} + \sum_{i=1}^{n} |\partial v/\partial x_i| \right)^2 \, ds \right)^{1/2}. \]

1 This work was supported in part by the Department of Energy through Grant No. DE-AC02-81 ER-10967 and the National Science Foundation through Grant No. DMS-8405689.
2 An alternative approach to the anisotropic plate has been given by Caillerie [3].

The boundary integral method for two-dimensional orthotropic materials

R.P. GILBERT & R. MAGNANINI
Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA

Received 27 May 1986

1. Introduction

This paper may be regarded as one in a sequence devoted to implementing a seminal idea that goes back to Fichera [8] for reducing the boundary value problems for higher order elliptic equations in the plane to an integral equation of the first kind. Subsequent to this Hsiao and McCamy [18, 28], applied the method of Fichera to the special problems

\[ \Delta^m u - \tau \Delta^{m-1} u = 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^2, \quad m = 1 \] or \[ 2. \]

\[ u = \phi \quad \text{on} \quad \partial \Omega \quad \text{for} \quad m = 1, \quad \nabla u = \psi \quad \text{on} \quad \partial \Omega \quad \text{for} \quad m = 2. \] (1.1)

They considered the situation where \( \Omega \) may be either an interior or exterior domain and the parameter \( \tau \) might be complex. The method of Fichera depends upon constructing a fundamental singularity \( S(x, y) \) for the partial differential equation, and using this to represent solutions as single layers. In the case (1.1) studied by Hsiao and McCamy, these simple layers are given by

\[ U^{(m)}(x, \psi) = \sum_{j=1}^{m} \int_{\Omega} \frac{\partial^{m+1} S(x, y)}{\partial x_j^{m+1} \partial y_j^{m+1}} \psi(y) \, ds, \] (1.2)

which led then to integral equations for the densities \( \psi(j = 1, \ldots, m) \) of the form [15]

\[ \int_{\Gamma} \sum_{j=1}^{m} K_{ij}(x, \psi(y) \, ds_j) = \phi_i(x) \quad \text{on} \quad \Gamma, \quad 1 \leq k \leq m, \] (1.3)

\[ K_{ij}(x, y) = \frac{\partial^{m+1} S(x, y)}{\partial x_j^{m+1} \partial y_j^{m+1}}, \quad 1 \leq j \leq m, \quad 1 \leq k \leq m. \]
A function theory for thin elastic shells

ROBERT P. GILBERT & YONGZHI XU
Department of Mathematical Sciences, University of Delaware, Newark, DE 19717, USA

Received 16 November 1987

Abstract. It is well known in the theory of elastic shells that a first order approximation using the shell thickness as an expansion parameter leads to the membrane theory of shells. The membrane equations have as solutions the generalized analytic functions. These functions have been exhaustively studied by Ilya N. Vekua [6], [7] and his students. R.P. Gilbert and J. Hile [3] introduced an extension of these systems to include elliptic systems of 2n equations in the plane and named the solutions of these systems generalized hyperanalytic functions.

It is shown in this paper that the next order approximation to the shell, which permits, moreover, the introduction of bending, may be described in terms of the generalized hyperanalytic functions. It is strongly suspected that the higher order approximations may also be described in terms of corresponding hypercomplex systems.

1. Notation and formulation

In this work we follow the notation used by Dikmen [1]: we view a shell as a three dimensional body, which we try to reduce to two dimensional consideration by introducing a suitable reference surface. A set of curvilinear coordinates \( \theta^i \) (i = 1, 2, 3) are chosen so that a reference surface within the shell may be represented by \( \theta^3 = 0 \). For purposes of defining our notation let \( \sigma \) be a surface embedded in \( \mathbb{R}^3 \) which we represent in the form \( r = r(\sigma) \). The vectors \( a_\sigma = r_\sigma = (ar)/(\sigma_\sigma) \) are the base vectors. The first fundamental form of the surface is given by \( a_\sigma a_\sigma = a^\alpha a^\beta a_\alpha a_\beta \) where the \( a_\alpha a_\beta = a_\alpha a^\gamma a^\beta_\gamma \) are the covariant components of the metric tensor. If the \( A_\alpha^\beta \) are the cofactors of the \( a_\alpha a_\beta \) we define the contravariant metric tensor as \( a^\alpha a^\beta = A_\alpha^\gamma A_\beta^\delta a^\gamma a^\delta \) where \( a \) is the determinant of \( A_\alpha^\beta \). The second fundamental form of the surface is defined through \( b_\sigma a_\sigma = b_\sigma a_\sigma a_\sigma a_\sigma = -dr^\sigma da_\sigma \), with \( b_\sigma = (a_\sigma \times a_\theta)/(\|a_\sigma \times a_\theta\|) \). The Christoffel symbols are given as usual by \( \Gamma_\alpha^\beta_\gamma = (a_\rho a_\gamma + a_\rho a_\sigma - a_\rho a_\alpha) \), and covariant differentiation is defined by

\[
T_\alpha^\gamma = T_\alpha^\gamma - \Gamma_\alpha^\gamma T_\sigma^\sigma.
\]  

The position vector of a generic point on the shell at time \( t \) is given by \( x = x(\theta^\alpha, \xi, t) \) where \( (\theta^1, \theta^2) \in \Omega \) is a point on the reference surface and \( \xi \in [0, h] \), in particular \( x(\theta^\alpha, 0, t) = r(\theta^\alpha, t) \). The methods of Dikmen [1] and

- Wavelet solutions for time harmonic acoustic waves in a finite ocean, J. Comp. Acoustics, 1(1)(1993), 31-61. (with W. Lin)


NONISOTHERMAL, NONNEWTONIAN HELE-SHAW FLOWS, 
PART II: ASYMPTOTICS AND EXISTENCE OF 
WEAK SOLUTIONS

R. P. GILBERT1 and P. SHI2

1Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, U.S.A.; and 
2Department of Mathematical Science, Oakland University Rochester, MI 48309, U.S.A.

(Received 9 November 1994; received for publication 10 February 1995)

Key words and phrases: Hele-Shaw flows, moving boundary problems, asymptotics, weak solutions.

1. INTRODUCTION

In this paper we first give a formal derivation of equations for the injection moulding starting from the basic equations for nonisothermal, nonNewtonian flow in a three-dimensional domain. Let \( \Omega = \Omega \times (-\varepsilon, \varepsilon) \) be a three-dimensional mold, where \( \Omega \) is a bounded domain in \( \mathbb{R}^2 \) with \( C^1 \) boundary, and \( 2\varepsilon \) represents the thickness of the mold. Using the method of asymptotic expansions about \( \varepsilon \) we arrive at a two-dimensional model that is independent of \( \varepsilon \).

The second part of the paper is devoted to the study of weak solutions to the resulting model. The present paper is a continuation of our earlier work [1].

We begin with the balance of linear momentum and the energy equations in \( \Omega_\varepsilon \):

\[
\rho \varepsilon \frac{D\mathbf{u}}{Dt} = \text{div} \sigma + \rho' f \tag{1.1}
\]

and

\[
\rho \varepsilon \left( \frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i} \right) = \frac{\partial}{\partial x_i} \left[ K \frac{\partial T}{\partial x_i} \right] + s_i d_i' \tag{1.2}
\]

where \( D\mathbf{u}/Dt \) is the material derivative of the velocity field \( \mathbf{u} = (u_1, u_2, u_3) \), \( \sigma = (\sigma_i^j) \) is the Cauchy stress tensor, \( e = (e_i^j) \) is the deviatoric part of \( \sigma_i^j \), \( \rho' \) is the density of the fluid, \( f \) is the volume force density, \( e \) is the specific heat, \( K \) is the thermal conductivity, and

\[
d' = (d_i') \quad \text{with} \quad d_i' = \frac{1}{2} \left( \frac{\partial e_i^j}{\partial x_j} + \frac{\partial e_j^i}{\partial x_i} \right) \tag{1.3}
\]

denotes the strain rate tensor. Summations with repeated indices are used. Note that the last term on the right hand side of (1.2) represents the heat generated by the deformation of the fluid under the action of the shear forces, the so called dissipation term. The physical
I visit China for the First Time, 1984

Nixon has opened up relations with China. I am in the first group to visit China. They have translated my book into Chinese. 7,500 copies are sold in China alone!

**Figure:** Acting like Nixon on the Great Wall

**Figure:** 7,500 copies!!!
Yongzhi Xu and I began our underwater research by extending the Colton-Kirsch theory for $\mathbb{R}^3$ to a waveguide.


- **Starting fields and far fields in ocean acoustics**, *Wave Motion* **11** (1989), 507-524 (with Xu, Yongzhi)
Dr. Robert Gilbert, an applied mathematician, and graduate assistant Yongzhi Xu (seated) work on a computer program that may help refine sonar, making the use of sound to detect submerged objects much quicker and more precise.
The Propagation Problem and Far-field Patterns in a Stratified Finite-depth Ocean

R. P. Gilbert and Yonghui Xu

Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, U.S.A.

In this paper we investigate the direct, or propagation, problem associated with the scattering of a 'plane wave' off a submerged body \( \Omega \). We assume that the body is contained in an ocean of finite, constant depth, which we refer to as \( \Omega \). The index of refraction, moreover, is assumed to be dependent only on the depth. Consequently, we are seeking solutions to the Helmholtz equation

\[ \Delta u + k^2 u = 0 \]  

in the region \( \Omega \), where \( u, \Delta u \) must satisfy certain boundary conditions on the ocean surface and bottom. For the purposes of this paper we assume that the ocean surface \( (z=0) \) is sound-soft, whereas the ocean bottom \( (z=-h) \) is sound-hard, i.e.

\[ u(x, y, 0) = 0, \quad \frac{\partial u}{\partial z}(x, y, 0) = 0. \]  

If the solution is decomposed into a sum of the incident wave \( u^i(x, y, z) \) and the scattered wave \( u^s(x, y, z) \), then on the ocean surface we have the additional boundary condition

\[ u(x, y, 0) = -\frac{\partial u}{\partial z}(x, y, 0) \]  

(1.2)

This problem was already considered by Gilbert and Xu [1], [5] for the case of an ocean with a constant index of refraction. It is clear that the above problem leads to a kind of exterior Dirichlet problem in a slab \( \Omega \). In order to make sure of the uniqueness of the problem, we should post a radiating condition in addition.

It is known, for instance, see [1] that the scattered wave \( u^s \) has the modal representation

\[ u^s = \sum_{n=1}^{\infty} \phi_n h(z)n(x, y) \]  

(1.3)

Received 3 April 1989

© 1989 by B. G. Teubner Stuttgart John Wiley & Sons, Ltd.
Armand Wirgin and I with Buchanan, Xu continued by developing the *Intersecting Canonical Domain Method for the Waveguide*.


A Personal Mathematical History

I wasn’t always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor

Indiana University Bloomington, Professor 1966-75

Analytic Methods in Mathematical Physics

My First Monograph

Generalized Hyperanalytic Function Theory

The Georgians

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University of Delaware, Unidel Professor 1975

Some Work in Elasticity, etc.

I visit China for the First Time, 1984

Research in Underwater Acoustics

The 65th Birthday Conference in Graz

The Cold War Ends: No More Underwater Acoustics

The Birth of ISAAC

I am 63 and I begin to work in Homogenization

My 70th Birthday Conference in Frejus, France

The 75th Birthday Conference in Orlando

Medical Mathematics

Figure: Armand Wirgin.
Identification of a 3D object in a shallow sea from scattered sound

Robert P. Gilbert, Thierry Scotti, Armand Wiggins, and Youngki S. XU

In this paper we investigate the unknown body problem in a wave guide where one boundary has a pressure release condition and the other an impedance condition. The method used in the paper for solving the unknown body inverse problem is the intersection canonical body approximation (ICBA). The ICBA is based on the Rayleigh conjecture, which states that every point on an illuminated body radiates sound from that point as if the point lies on its tangent sphere. The ICBA method requires that an analytical solution be known exterior to a canonical body in the wave guide. We use the sphere of arbitrary centre and radius in the wave guide as our canonical body. We are lead then to analytically computing the exterior solution for a sphere between two parallel plates. We use the ICBA to construct solutions at points ranging over the suspected surface of the unknown object to reconstruct the unknown object using a least-squares matching of computed, acoustic field against the measured, scattered field.

1. INTRODUCTION

In this paper we investigate the localization and identification of a three-dimensional object in an acoustic wave guide by inversion of the scattered wave field. This inverse problem is non-linear in terms of position and shape parameters of the object and fundamentally ill posed. Furthermore, because of the multiple scattering of sound between the object and the top and bottom boundaries of the sea, the associated direct problem is much more complicated than for a body in space. This problem has important practical marine applications such as identification of mineral deposits, submarines, submersed navigational obstacles and wreckage.

Identification of objects in an acoustic wave guide
inversion II: Robin–Dirichlet conditions

Doo-Sung Lee and R. P. Gilbert

SUMMARY

In this paper we investigate the unknown body problem in a wave guide where one boundary has a pressure release condition and the other an impedance condition. The method used in the paper for solving the unknown body inverse problem is the intersection canonical body approximation (ICBA). The ICBA is based on the Rayleigh conjecture, which states that every point on an illuminated body radiates sound from that point as if the point lies on its tangent sphere. The ICBA method requires that an analytical solution be known exterior to a canonical body in the wave guide. We use the sphere of arbitrary centre and radius in the wave guide as our canonical body. We are lead then to analytically computing the exterior solution for a sphere between two parallel plates. We use the ICBA to construct solutions at points ranging over the suspected surface of the unknown object to reconstruct the unknown object using a least-squares matching of computed, acoustic field against the measured, scattered field.

Correspondence to: R. P. Gilbert, Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, U.S.A.
E-mail: gilbert@math.udel.edu
Contract/grant sponsor: Alexander von Humboldt Foundation
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Published online 8 December 2005 in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/mma.687
MOS subject classification: 35R30; 73D50
Inverse problems in a waveguide are numerically more difficult than in $\mathbb{R}^3$ because of the attenuation of certain modes.

Figure: The original object, a spindle

Figure: The reconstructed spindle
Figure: The original object, a torus

Figure: The reconstructed torus
The $65^{th}$ Birthday Conference in Graz

Some of the people at my $65^{th}$ birthday conference in GRAZ, Austria.
A Personal Mathematical History

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Indiana University Bloomington, Professor 1966-75

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University of Delaware, Unidel Professor 1975

Some Work in Elasticity, etc.

I visit China for the First Time, 1984

Research in Underwater Acoustics

Figure: Elena Obalishvili at Graz

Figure: Okay Celebi
The Cold War Ends: No More Underwater Acoustics

We were happy to have funding from NOA, NSF and ONR for another 15 years for our research on Inverse Problems in an underwater environment. We collected our research in the following (my fifth monograph):

![MARINE ACOUSTICS](image-url)
The Birth of ISAAC

At the Hong Kong Complex Analysis meeting in 1997 I floated the idea of starting an international society devoted to real and complex analysis. The idea was strongly endorsed by the attendees. In order to get the thing moving I agreed to hold the first meeting at the University of Delaware. The turnout was impressive, approximately 400 attendees and ISAAC the International Society for Analysis, Applications and Computation was born. We decided to elect officers, a board and to approve a constitution. Subsequent meetings were held at Fukuoka University Japan, the Free University Berlin, University of Catania, the Mideast Technical University Ankara, York University Toronto and last year at Imperial College London where we had 600 attendees. Pierre Lions gave a plenary talk at this meeting along with Sir John Ball and Günter Uhlmann. This next summer the meeting will be at the Steklov Institute Moscow. All are welcome to attend.
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7th ISAAC Congress
Imperial College London
July 13-18, 2009

International Society for Analysis, its Applications and Computation

Confirmed Plenary Speakers
Sir John Ball (Oxford)
Nicole Lerner (Paris)
Paul Malliavin (Paris)
Vladimir Maz’ya (Liverpool)
Bert-Wolfgang Schulze (Potsdam)
Masahiro Yamamoto (Tokyo)

Public Lecture on Nonlinear PDEs (Monday 13 July)
Pierre-Louis Lions (Paris)

Sessions
I.1. Complex variables and potential theory
I.2. Differential equations: complex methods, applications
I.3. Complex-analytical methods in applied sciences
I.4. Value distributions of functions
I.5. Clifford and quaternion analysis
I.6. Methods in Clifford- and Cayley-
I.7.1. Toeplitz operators and applications
I.7.2. Reproducing kernels
I.7.3. Integral transforms
I.7.4. Spaces of differentiable functions
I.7.5. Analytic function spaces
I.7.6. Spectral theory
I.7.7. Pseudo-differential operators
I.7.8. Control and optimisation of evolution equations
I.7.9. Nonlinear PDE
I.7.10. Asymptotic and multiscale analysis
I.7.11. Inverse problems
I.7.12. Coercivity and functional inequalities
I.7.13. Dynamical systems
I.7.15. Mathematical biology
I.7.16. Others

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Contact and Further Information
Michael Ruzhansky, Department of Mathematics, Imperial College London,
180 Queen's Gate, London SW7 2AZ, UK
http://www.isaac2009.org

Figure: The announcement of the London meeting.
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Medical Mathematics

Figure: Lee Lorch, Victor Berenkov and Heinrich begehr.
I began to work in Homogenization on my sabbatical to Lyon and St. Etienne in 1995. This work was supported by ONR, NATO and NSF. My first paper is with Grisha Panasenko. My second paper is with Andro Mikelic.

Figure: A. Bourgeat, G. Panaseko and the Gilberts
Homogenizing the acoustic properties of the seabed: Part I, Nonlinear Analysis 40, 185-212, (2000), (with A. Mikelić) (45)


Vibration of two bonded composites: effects of the interface and distinct periodic structures, Int. J. Solids and Structures, 40 (2003), 3177-3193, (with Michael Harik and Alexander Panchenko)
Homogenizing the acoustic properties of the seabed: Part I

R.P. Gilbert, A. Mikelić

*Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA
bAnalyse Numérique, CNRS UMR 5583, Bât. 101, Université Lyon 1, 43, Bd du 11e novembre, 69622 Villeurbanne Cedex, France

Keywords: Monophasic Biot law; Poroelastic medium; Homogenization

1. Introduction

One of the pressing problems in underwater acoustics today is formulating and then solving a model for interaction of acoustic waves in a shallow ocean with the seabed. Shallow-water/seabed waveguide, direct and inverse wave propagation problems are ubiquitous in applied science and technology. One such application is for inverse imaging of objects submerged in the ocean or the seabed. As much of the acoustic energy passes into the seabed, this imagery is possible only if the sea environment (water, sediment, interfaces), in the absence of the object, is properly characterized beforehand.

This means that a suitable model of the sediment and of propagation of sound therein must be developed and a method be proposed for solving the inverse problem of the identification of the mechanical parameters involved in this model. This model, as well as the sediment parameter and object identification scheme, must be able to take into account sound speed and density variations in the water as well as the behavior of sound in the seabed.

In general, either an acoustic pulse, or a monochromatic signal with frequency \( \omega \) is used. Consequently, not only acoustic signals with acoustic frequencies spread about a central frequency, but time-harmonic solutions are of interest. There have been several acoustic models of the seabed [9,10,16]; however, the primary one in usage goes back

---

\( \frac{362.8 \times 272.1}{1080 \times 1080} \)

**Figure:** Chez le maison de Andro Milić à St. Etienne.
Homogenizing the Acoustic Properties of the Seabed, Part II

THIERRY CLOPEAU
Analyse Numérique, CNRS UMR 5585
Bât. 101, Université Lyon 1
43 Bd. du onze novembre, 69622 Villeurbanne Cedex, France

J. L. FERRÉN
Departamento de Matemática Aplicada
Universidad de Santiago de Compostela
15706 Santiago de Compostela, Spain

R. P. GILBERT
Department of Mathematical Sciences
University of Delaware
Newark, DE 19716, U.S.A.

A. MIKELIĆ
Analyse Numérique, CNRS UMR 5585
Bât. 101, Université Lyon 1
43 Bd. du onze novembre, 69622 Villeurbanne Cedex, France

(Received and accepted May 2004)

Abstract—We undertake a rigorous derivation of the diphonic Biot’s law, describing small deformations of a seabed of the characteristic size L0/\varepsilon and containing a pore structure of the characteristic size \varepsilon. The solid part of the seabed (the matrix) is elastic and the pore contains a viscous fluid. The fluid is supposed incompressible or slightly compressible. In this case, the contrast of property is of order \varepsilon^2, i.e., the normal stress of the elastic matrix is of the same order as the fluid pressure. We suppose a periodic matrix and obtain the a priori estimate. Then we let the characteristic size of the inhomogeneities tend to zero and pass to the limit in the sense of the two-scale convergence. The obtained effective equations represent a two-scale system for three velocities and two pressures. We prove uniqueness for the homogenized two-scale system. Then we introduce several auxiliary problems and obtain a problem without the fast scale. This new system is diphonic and corresponds to the diphonic Biot’s law already observed in papers by Biot. In the effective equations, it is possible to distinguish the velocities of the fluid and the solid part, respectively. The effective stress tensor contains an instantaneous elasticity tensor and there are double porosity terms. We give a detailed study of the effective equations and compare them with the original Biot’s poroelasticity equations. © 2004 Elsevier Science Ltd. All rights reserved.

1. Introduction

In this paper, we continue the study of the acoustic behavior of the seabed. In [1], the modeling of the interaction between an elastic matrix and an incompressible fluid or slightly compressible fluid, this work was partially supported by NATO Research Grant CRIS 975201. Part of the work by J. L. Ferré is supported by a grant from the Xunta de Galicia, Spain.

0305-1978/04 - see front matter © 2004 Elsevier Science Ltd. All rights reserved. Type set by Agfa-Topzeta.

Acoustics of a Stratified Poroelastic Composite

R. P. Gilbert and A. Panchenko

Abstract. In this paper we discuss the acoustic boundary layer problem for a poroelastic seabed submerging into a liquid half space. The problem is addressed using the method of homogenization where the microscopic equations are modelled after Biot and Keller [5], Levy [10], and Papanastasiou [13]. A difference in our approach is that we do not consider the viscosity coefficients to be dependent on the pore size. To achieve continuity of displacement and stress at the interface to an arbitrary asymptotic order, we introduce correctors of two different types on each side. Then corrections of different types are matched across the interface.

Keywords: Homogenization, asymptotic expansions, boundary layers, composite materials

AMS subject classifications: 74 Q10, 35 Q35

1. Introduction and Remarks

In this paper we discuss the vibrational motion of a porous medium whose pore space is saturated with fluid. The porous medium we propose to study is formed by a periodic arrangement of the pores into cells. The vibrational motion is assumed to be stimulated acoustically by a signal whose wave length is \lambda. For an averaging procedure to work, we need the wavelength to be large compared to a typical cell size \ell. Assuming in addition that \lambda is comparable to the characteristic macroscopic size \ell of the problem and the fluid phase is incompressible, one can classify different homogenized models, as was done in [2] heuristically, and justified rigorously in [7]. In these works, four different types of possible macroscopic behavior are listed:

- Model I: The acoustics of a fluid in a rigid porous matrix regime. This case was considered previously by Gilbert and Papanastasiou [6].
- Model II: Diphasic macroscopic behavior of the fluid and solid matrix. This case is considered using the methods of two-scale convergence in [7].
- Model III: Monophasic elastic macroscopic behavior. This case is also discussed in [7].
- Model IV: Monophasic viscoelastic macroscopic behavior.

Model II, the diphasic case corresponds to the Biot model [3, 4].

In this paper, we allow the fluid to be compressible and do not assume that \lambda is comparable to \ell. Thus the model developed in the paper will also work for \lambda large.

Both authors: University of Delaware, Dept. Math. Sci., Newark, DE 19716, U.S.A.
Authors were partially supported by the NSF grant DMS-9420359.

ISSN 0122-2064 / $ 3.50 © Reidelmann Verlag Berlin
I begin to work with two of my students, Alex Panchenko and Yvonne Ou, on homogenization.

Figure: Yvonne Ou
A Personal Mathematical History

I wasn’t always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65 Assistant Professor and Associate Professor

Indiana University Bloomington, Professor 1966-75

Analytic Methods in Mathematical Physics

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The 75th Birthday Conference in Orlando

Medical Mathematics

Figure: Gaetano and Mathilda Fichera with friends.

Figure: Heinrich Begehr, Wolfgang Wendland and friends.
My 70th Birthday Conference in Frejus, France

I was lucky that Armand Wirgin got CNRS funding from the French Government for my 70th birthday celebration in FREJUS on the Cote d’Azur. This resulted a collection of very nice papers.
A Personal Mathematical History

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Figure: Armand Wirgin.
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Medical Mathematics

Figure: Mazya, Jenny and Lenny Schwarz

Figure: Lin Wei and Andro Mikelic
A Personal Mathematical History
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Generalized Hyperanalytic Function Theory
The Georgians
Free University Berlin and the Hahn-Meitner Institute 1974-75
University of Delaware, Unidel Professor 1975
Some Work in Elasticity, etc.
I visit China for the First Time, 1984
Research in Underwater Acoustics
The 65th Birthday Conference in Graz
The Cold War Ends: No More Underwater Acoustics
The Birth of ISAAC
I am 63 and I begin to work in Homogenization
My 70th Birthday Conference in Frejus, France
The 75th Birthday Conference in Orlando
Medical Mathematics

Figure: Klaus Hackl
Figure: Lou Fishman
A Personal Mathematical History

I wasn’t always a mathematician

Institute for Fluid Dynamics and Applied Mathematics 1961-65
Assistant Professor and Associate Professor

Indiana University Bloomington, Professor 1966-75

Analytic Methods in Mathematical Physics
My First Monograph
Generalized Hyperanalytic Function Theory
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Medical Mathematics

Figure: The Mazyas and me

Figure: Heinrich Begehr and Franco Nicolosi
A Personal Mathematical History

I wasn’t always a mathematician

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Figure: Richard Delanghe and Jerry Hile

Figure: Jim Buchanan and Lenny Schwarz
Some more recent papers:


AN OBSTACLE PROBLEM IN NON-ISOTHERMAL AND NON-NEWTONIAN HELE-SHAW FLOWS

R.J. Gilbert and F. Peng
Department of Mathematical Sciences
University of Delaware
Newark, DE 19716, USA

ABSTRACT: In this paper, we prove the existence of a solution to new type of nonlinear system, where a nonlinear, variational inequality is coupled with a nonlinear equation.

AMS (MOS) Subject Classification: 35J20, 35K65, 35R35, 35B45, 35J55, 35E20

1. INTRODUCTION

A Hele-Shaw flow occurs when a slow viscous incompressible fluid moves between two slightly separated parallel plates. Several models have been proposed by Biffoner-Taylor [29], Richardsen [39], Elliott and Janovsky [7], Greenspan and Gilbert [3], Beaucy and Gilbert [4], Gilbert and Wu [15], Gilbert and Shi [18], etc. Some models are linear while others are nonlinear. Some deal with one partial differential equation whereas others include a system of two partial differential equations. To investigate these models, the theoretical methods of functions: Richardsen [29], Gilbert and Wu [15], Howison et al [21], Tavizer [36], Xin [29], partial differential equations: Greenspan and Gilbert [3], Beaucy and Gilbert [4], Gilbert and Shi [15], Gilbert and Peng [8], variational inequalities: Elliott and Janovsky [7], Greenspan and Gilbert [3], Beaucy and Gilbert [4], Gilbert and Shi [18], etc. have been employed.

The formulation of moving boundary problems in terms of a variational inequality reduces the complexity in that there is no explicit mention of the free boundary. However, in the study of existence of solutions, compactness, and regularity, the free boundary becomes explicit. Beaucy and Gilbert [3], Beaucy and Gilbert [4], Beaucy and Gilbert [4], Shi [18].

Received February 17, 2002

RICHARDS, R.P. Gilbert and F. Peng
Department of Mathematical Sciences
University of Delaware
Newark, DE 19716, USA

Effective Acoustic Equations for a Two-Phase Medium with Microstructure

R. P. GILBERT
Department of Mathematical Sciences
University of Delaware
Newark, DE 19716, U.S.A.
A. PANCHENKO
Department of Mathematics
Washington State University
Pullman, WA 99164, U.S.A.

Abstract—We study acoustic wave propagation in a two-phase medium in which the solid phase is a linear elastic medium, and the fluid phase is assumed to be a compressible Newtonian incompressible fluid. Assuming that properties of the medium change rapidly on the scale of the interface, we analyze the linear microscopic Helmholtz equations and show that they can be linearized when a length scale is introduced. Using a variant of Tatars' method of modulating test functions, we derive effective acoustic equations which turn out to be variational. In order to treat discontinuous materials occurring in the two-phase medium, we apply an approach to studying geometry of a two-phase medium with length scale separation. Our approach is not based on probabilistic considerations. Instead, we use a probabilistic approach to studying the microstructure of the medium. © 2004 Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Averaging solid-fluid composites with instantaneous memory typically produces history dependent viscoelastic effective equations [1–4], while the specific mechanism of the microscopic energy dissipation is reflected in the choice of the constitutive functions. Motivated by this, we have attempted to lay a fairly general theoretical foundation for derivation of the viscoelastic models for acoustics of two-phase materials with microstructure.

The size of a typical microstructural element (e.g., a pore cavity) is characterized by a so-called microscopic length scale γ which is much smaller than the sample size. The microscopic length scale γ is introduced by assuming that material properties are of the form f(x, γy, z), where γ = γ y is the so-called fast variable. Using techniques of homogenization, the actual highly inhomogeneous material can be replaced by a quasi-uniform effective medium with the material properties of the form f(γz, γy, z), variations of which are significant only over distances comparable to the size of the whole sample.

Previously, work in this field [1–4] was limited to analysis of linear microscopic equations. In addition, the authors of [2–4] assumed that the propagating wave is time-harmonic, which reduces

The work of these authors was supported in part by ONR Grant N00014-02-00833.
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A Personal Mathematical History

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Medical Mathematics

A Two-Dimensional Nonlinear Theory of Anisotropic Plates

R. P. GILBERT

Department of Mathematical Sciences
University of Delaware
Newark, DE 19716, U.S.A.
gilbert@math.udel.edu

T. S. VASHAKMDZE

Javakhishvili, Tbilisi State University
Chavchavadze Av. 1, Tbilisi, Georgia
vashak@tsu.gep.edu.ge

(Received December 1999; accepted January 2000)

Abstract—We develop a two-dimensional theory of nonlinear, anisotropic plates using the variational
approach of Clarlet. Under very general conditions on the elastic coefficient for the linear
case, it is shown that there is a unique solution to the associated variational problem. Moreover,
refined theories are constructed for these systems. With regard to nonlinear elastic plates, we con-
sider Filon's nonlinear, anisotropic model. Dynamical, nonlinear models are also considered. © 2000
Elsevier Science Ltd. All rights reserved.

1. INTRODUCTION

Constructing two-dimensional models without a priori assumptions concerning geometrical and
physical characteristics are due, in particular, to the investigations of [1-6]. These works have
a particular meaning for the development of a mathematical theory of anisotropic elastic plates.
Evidently, the creation of a rigorous theory of elastic plates and shells also have application to
problems of continuum mechanics.

In this direction, recently Gilbert and his coworkers published the series of works [7-13] devoted
to the study of direct and inverse problems in the acoustic of shallow oceans with poroelastic
sediments. As the interest in poroelastic, anisotropic sediments for ocean acoustics is well estab-
lished, we consider some extensions of the results of [14,15], for anisotropic elastic homogeneous
plates with one elastic symmetry plane. Our goal is to construct two-dimensional models, in
terms of the so-called engineering coefficients: Young's moduli (E_y), Poisson ratios (v_y), and
the shear moduli (G_y) (see [14]), without having to resort to asymptotic methods, as has been the
case with the authors [1,3,16].

This work is the first devoted to the creation a rigorous nonlinear theory for ocean acoustic
problems with poroelastic sediments.

For clarity and brevity, we adopt notations used in [14,17].

This work supported by the National Academy of Sciences (NAS) and U.S. Recipients of Collaboration in Basic
Science and Engineering (COBASE), 1999, Long-Term Fellowships to Host Colleague.

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ISSN 0232-2064 / $ 2.50 © Heldermann Verlag Berlin
Yvonne Ou and Alex Panchenko decide to do something about my 75th birthday.

I really like parties!
I wasn’t always a mathematician

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Medical Mathematics

Figure: D. Khavinson, J. Ryan and G. Auchmutty at my 75th
A Personal Mathematical History

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Figure: Armand Wirgin and me at my 75th
Since the cold war was over and there was no more funding for the type of problems we were interested in we decided first to look into another poroelastic material, namely bone. This has led to an active research program and funding from NSF for the International Program, NATO and the NSF Bio-Math Program. We have been interested in determining the rigidity of bone, modeling the disease osteoporosis and medical-mathematics in general.
A Personal Mathematical History

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Figure: The Representative Volume Element

Figure: Computational procedure

Global FE Analysis - Macrolevel

1. $\tilde{\epsilon}$ is given

2. Calculation of $\mathbf{x}$ and $\tilde{\mathbf{u}}$ as a solution of BVP on the RVE.
   (Standard FE-principles in $\mathcal{B}$.)

3. Calculation of $\bar{\sigma}$ as a volume average:
   $$\bar{\sigma} = \frac{1}{V} \int_{\mathcal{B}} \sigma dV, \quad \bar{\mathbf{C}} = \frac{\partial \bar{\sigma}}{\partial \bar{\epsilon}}$$

Global FE Analysis - Macrolevel

↓ $\tilde{\epsilon}$

↓ $\bar{\sigma}, \bar{\mathbf{C}}$
I begin to work with Klaus Hackl again, this time on Quantitative Ultrasound.

Figure: Klaus and Martrina Hackl with Nancy.
Papers in bio-mathematics:


Determination of the parameters of cancellous bone using high frequency acoustic measurements

James L. Buchanan*, Robert P. Gilbert

* Mathematics Department, United States Naval Academy, Annapolis, MD, United States

1. Introduction

Cancellous bone is a two component material consisting of a calcified bone matrix with interstitial fatty marrow. Faced with the clinical need to understand better the effects of aging and disease on the elastic and strength properties of trabecular bone, researchers have begun to investigate whether factors such as a architecture and tissue quality can account for 20%-40% of unexplained variance in these mechanical properties [10]. Quantitative ultrasound QUS attempts to interrogate the bone structure and determine whether the subject has osteoporosis; however, QUS is still in its infancy, Chaffai et al. [6]. It relies on assessment of two fundamental parameters: broadband ultrasound attenuation BUA and speed of sound SOS.

The magnitude and frequency dependence of ultrasound attenuation is a complicated function of the composition and the micro-architecture of the propagating medium. However, one needs to note a lack of theoretical model that describes how the attenuation is influenced by properties of bone. Hodgkinson et al. [7] studied the ability of ultrasound velocity to predict the Young’s modulus of elasticity of cancellous bone. QUS measurements have been successfully used recently to predict fracture risk in osteoporosis. This technique is now incorporated into commercial devices enabling attenuation to be measured. Hence mathematical models of poroelastic media have been applied to the study of bone architecture. Indeed, McKelvie and Palmer [9], Williams [11], and Hosokawa and Otani [8] discuss the application of the Biot model for a poroelastic medium.
Computing porosity of cancellous bone using ultrasonic waves, II: The muscle, cortical, cancellous bone system

Robert P. Gilbert, Ying Liu, Jean-Philippe Groby, Erick Ogam, Armand Wirgin, Yongzhi Xu

1. Introduction

Cancellous bone consists of a trabecular matrix with an interstitial blood–marrow fluid. Osteoporosis is characterized by a decrease in strength of this bone matrix. Currently, bone mineral density (BMD) is the gold standard for in vivo assessment of fracture risk of bones and is measured using x-ray absorptiometric techniques [1]. However, only 70–80% of the variance of bone strength is accounted for by bone density [2]. At the brittle of bones depends on more factors than bone density, biologists believe that quantitative ultrasound techniques (QUT) can provide an important new diagnostic tool. Moreover, in contrast to x-ray densitometry, ultrasound does not involve the mineralized tissue, and its implementation is relatively inexpensive. It would be of enormous clinical advantage if an accurate method could be developed using ultrasound interrogation to determine whether one had osteoporosis. The intention of this research is to eventually produce an accurate clinical procedure for determining the bone density and other bone parameters describing bone brittleness.

Since the loss of bone density and the destruction of the bone microstructure is evident in cancellous bone, it is natural to consider the possibility of developing accurate ultrasound models for the sonification of cancellous bone. In this paper, we consider the possibility of determining the bone density of cancellous bone in vivo by using a simple one-dimensional model of a muscle–cortical bone–cancellous bone sample. If it is possible to accurately determine whether an in vivo sample is osteoporotic, research in this direction merits further investigation. Moreover, if a higher dimensional model also leads to an accurate prediction of bone density, this would warrant laboratory testing in vivo on biological samples. More specifically, we determine whether a sonified bone sample is osteoporotic by measuring its refracted acoustic, i.e., the acoustic field is measured when the sample is exposed to impulses from a transducer. From this data the bone parameters can be determined.
Application of the multiscale FEM to the modeling of cancellous bone

Sandra Ilie - Klaus Hackl - Robert Gilbert

Received: 6 February 2009 / Accepted: 15 June 2009 / Published online: 1 July 2009
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Abstract This paper considers the application of multiscale finite element method (FEM) to the modeling of cancellous bone as an alternative to Bloch’s model, the main intention of which is to decrease the extent of the necessary laboratory tests. At the beginning, the paper gives a brief explanation of the multiscale concept and thereafter focuses on the modeling of the representative volume element and on the calculation of the effective material parameters, including an analysis of their change with respect to increasing porosity. The latter part of the paper concentrates on the macroscopic calculations, which is illustrated by the simulation of ultrasonic testing and a study of the attenuation dependency on material parameters and excitation frequency. The results endorse conclusions drawn from the experiments: increasing excitation frequency and material density cause increasing attenuation.

Keywords Multiscale FEM · Attenuation · Cancellous bone · Homogenization

1 Introduction

In recent times, the investigation of the effective properties of cancellous bone has been especially intensive as it may lead to the early detecting of osteoporosis. This pathological process manifests itself through increasing bone resorption and decreasing bone production. In its late stage, even entire walls of the solid bone frame disappear leading to abrupt decrease of material strength. The laboratory measurements show that during the process porosity increases from 72% up to 95% causing the density to change from 1,200 to 1,000 kg/m$^3$.

For the investigation of cancellous bone different approaches are developed, some of them being of experimental character, the other ones focusing on developing a convenient mathematical or mechanical model. Among the experimental methods the dual X-ray absorptiometry (DXA) and the quantitative ultrasonic (QUS) technique are the ones mostly used, but the latter has a few important advantages; while the DXA is convenient only for investigation of bone mineral density (BMD), the QUS technique yields data on the speed of sound (SOS) and broadband ultrasonic attenuation (BUA), two parameters strongly related to the microscopic structure of trabecular bone and significantly influencing its strength (Hosokawa and Osumi 1997; Zysset et al. 1999; Burkman et al. 2000; Bowy et al. 2004; Laugier et al. 1994). Moreover, in contrast to X-ray technology, ultrasound does not ionize the tissue and its implementation is relatively inexpensive.

Among the analytical solutions, those based on Bloch’s theory (Bloch 1956a,b) are certainly established as the leading ones. The shortcoming of these methods is its dependence on many material parameters which have to be determined experimentally. An analytical approach to the investigation of these parameters and especially of the accuracy up to which they can be determined, is presented in the works of Buchanan et al. (2004) and Buchanan and Gilbert (2006).

Finally, the rush development of computer technology gave a new aspect to the bone modeling where the application of numerical solvers plays an important role. Here also, two groups of methods deserve to be mentioned: the finite difference time domain (FDTD) technique and the finite...
Acoustic propagation in a random saturated medium: The monophasic case

Robert P. Gilbert*, Alexander Panchenko and Ana Vasilic†

Communicated by W. Wendland

We study the problem of derivation of an effective model of acoustic wave propagation in a two-phase, non-periodic medium modeling a fine mixture of linear elastic solid and a viscous Newtonian fluid. Bone tissue is an important example of a composite material that can be modeled in this fashion. We employ techniques of stochastic two-scale convergence in the mean to pass to the limit. The ratio of the macroscopic length scale and a typical size of the microstructural inhomogeneity is a small parameter of the problem. We employ periodic geometries to the case of a stationary random, scale-separated microstructure. The ratio of the macroscopic length scale and a typical size of the microstructural inhomogeneity is a small parameter of the problem. We employ periodic geometries to the case of a stationary random, scale-separated microstructure. We extend known homogenization results for examples of composite materials that can be modeled in this fashion. We extend known homogenization results for stochastic microstructures to the case of a stationary random, scale-separated microstructure. We are interested in the effective model for acoustic wave propagation in a two-phase, non-periodic medium modeling a fine mixture of linear elastic solid and a viscous Newtonian fluid. Bone tissue is an important example of a composite material that can be modeled in this fashion. We employ techniques of stochastic two-scale convergence in the mean to pass to the limit. The ratio of the macroscopic length scale and a typical size of the microstructural inhomogeneity is a small parameter of the problem. We employ periodic geometries to the case of a stationary random, scale-separated microstructure.
Mathematics Genealogy Project

Robert Pertsch Gilbert

Ph.D. Carnegie Mellon University 1958

Dissertation: *Singularity of the Three-Dimensional Harmonic Functions*

Advisor: Zeev Nehari

Students:

Click [here](#) to see the students listed in chronological order.

<table>
<thead>
<tr>
<th>Name</th>
<th>School</th>
<th>Year</th>
<th>Descendants</th>
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<td>Satish Bhatnagar</td>
<td>Indiana University</td>
<td>1974</td>
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<tr>
<td>Patrick Brown</td>
<td>Indiana University</td>
<td>1973</td>
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<td>Ronald Brown</td>
<td>Indiana University</td>
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<td>James Buchanan</td>
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<td>Ming Fang</td>
<td>University of Delaware</td>
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<td>Gerald Hile</td>
<td>Indiana University</td>
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<td>Khaldoun Khashanah</td>
<td>University of Delaware</td>
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<td>Dean Kukral</td>
<td>Indiana University</td>
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<td>Pingqian Li</td>
<td>University of Delaware</td>
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<td>Zhongyan Lin</td>
<td>University of Delaware</td>
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<td>Chi Lo</td>
<td>University of Maryland College Park</td>
<td>1966</td>
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<td>Rolando Magnanini</td>
<td>Università di Florence</td>
<td>1969</td>
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<td>Edward Newberger</td>
<td>Indiana University</td>
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<td>Miao-jung Qu</td>
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<td>Alexander Panchenko</td>
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<td>P. Ramakutty</td>
<td>Indiana University</td>
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<td>Robert Ronsesee</td>
<td>University of Delaware</td>
<td>2007</td>
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<td>Peter Lai Sheng Shi</td>
<td>University of Delaware</td>
<td>1988</td>
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<td>Ana Vasilic</td>
<td>University of Delaware</td>
<td>2009</td>
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<td>Yongzhi Xu</td>
<td>University of Delaware</td>
<td>1990</td>
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<tr>
<td>Ningyi Zhang</td>
<td>University of Delaware</td>
<td>2007</td>
<td></td>
</tr>
</tbody>
</table>

According to our current on-line database, Robert Gilbert has 21 students and 28 descendants.

**Figure:** 21 PhDs and 28 descendants to-date.
I, with the cooperation of Philippe Guyenne, am advising four PhD students presently

1. Ying Liu
2. Patrick Rowe
3. Michael Shoushani
4. Jing Li

We (Guyenne, Liu and myself) are working on a mathematical model for the osteoblastic cell. It is our objective to develop a variant of the Virtual Cell such as exists at the Medical School of the University of Connecticut. The purpose of such a cell is to suggest biological experiments and to pose research projects for cell biology. We are discussing the modeling of the cell with Anja Nohe in Biochemistry.
Figure: Looking backwards.
Chapter 9
Homogenization Theories and Inverse Problems

Robert P. Gilbert and Ana Vasilic

Abstract Various approaches are presented for modelling the acoustic response of cancellous bone to ultrasound interrogation. As the characteristic pore size in cancellous bone is much smaller than a typical bone sample, there is a clear scale separation (micro versus macro). Thus, our modelling methods are mainly based on homogenization techniques and numerical upscaling. First, we consider the so-called direct problems and present models for both periodically perforated domain and a domain with random distribution of pores, as well as nonlinear model with a shear-thinning viscoelastic material emulating the blood-marriage mixture. A numerical procedure is given for the upsampling of a biphasic mixture using different trabecular thicknessess and various frequencies for the ultrasound excitation. Finally, the results of a quite accurate two-dimensional inversion for the Biot parameters is presented. Further details for these different problems are amply described in the literature cited in the bibliography.

9.1 Introduction
Osteoporosis is characterized by a decrease in strength of the bone matrix. Currently, bone mineral density (BMD) is the gold standard for in vivo assessment of fracture risk of bones and is measured using X-ray absorptiometric techniques [18]. However, only 70–80% of the variance of bone strength is accounted for by bone density. As the brittleness of bone depends on more factors than bone density, biologists

R.P. Gilbert E Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, USA e-mail: gilbert@math.udel.edu
A. Vasilic E Department of Mathematical Sciences, UAE University, PO. Box 17551, Al Ain, Abu Dhabi, UAE e-mail: vasilic@uaeu.ac.ae

Figure: A chapter from Bone Quantitative Ultrasound, Springer, 2011
A Personal Mathematical History

I wasn’t always a mathematician

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Medical Mathematics

Figure: Here’s looking at you! Catania, Conference on Nonlinear PDE