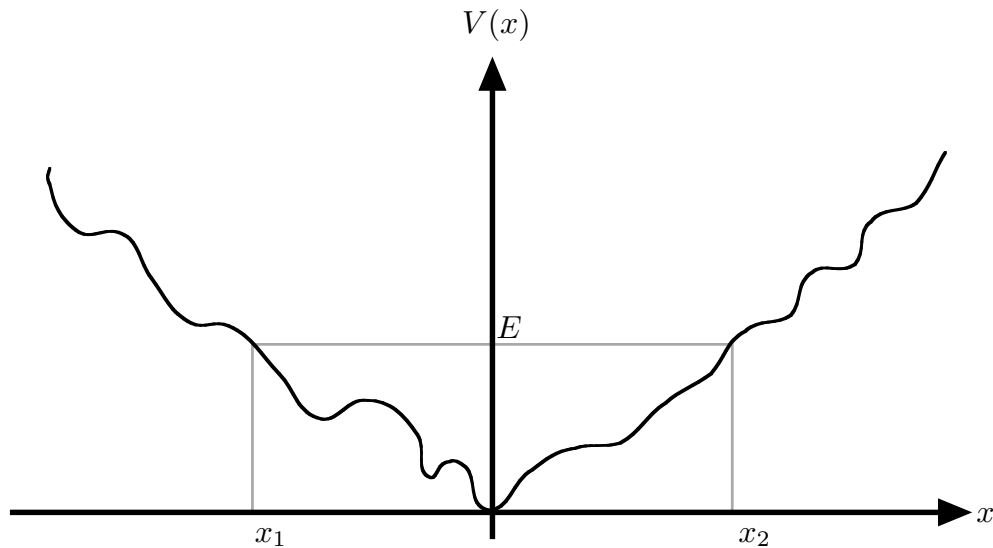


## Homework Set 8

Read sections 6.1–6.4 and sections 10.4–10.6.

### Turning Points



1. Consider the following problem:

$$u'' + \lambda^2[E - V(x)]u = 0, \quad -\infty < x < \infty, \quad \lambda \rightarrow \infty,$$

where  $V(x)$  is as shown above. We wish to analyze bounded solutions of this operator.

- (a) (4 points) Find the outer solutions to the problem. (*Hint: You need not repeat all the analysis; just make sure have all the variables defined in an appropriate way.*)
- (b) (5 points) Using the matching formulas given in class, construct the full solution to the problem.
- (c) (2 points) Show that a solution can exist for  $x \in [x_1, x_2]$  only for values  $E_n$  of  $E$  given by

$$\int_{x_1}^{x_2} \sqrt{E_n - V(x)} dx = \frac{(n + 1/2)\pi}{\lambda}.$$

2. Consider the following problem:

$$w'' - k^2x(x^2 + 1)^2w = 0, \quad x \geq -1, \quad w(-1) = 0, \quad w(\infty) = 0, \quad k > 0.$$

- (a) (5 points) Find the outer solutions to the problem. (*Hint: You need not repeat all the analysis; just make sure have all the variables defined in an appropriate way.*)
- (b) (5 points) Using the matching formulas given in class, construct the full solution to the problem.
- (c) (3 points) Show that a nontrivial solution can exist only for values of  $k$  approximately equal to

$$\frac{21(4n - 1)\pi}{80}, \quad n \in \mathcal{Z}^+.$$

### Watson's Lemma

3. Consider the function

$$f(z) = \int_0^\infty \frac{e^{-zu}}{1 + u^4} du.$$

- (a) (2 points) Find the region of analyticity for  $f(z)$ .
- (b) (4 points) Find its complete asymptotic expansion as  $z \rightarrow \infty$  in this region.
- (c) (7 points) Show that

$$g(z) = \frac{\pi(1 + i)}{2\sqrt{2}} \exp\left(ze^{3\pi i/4}\right) - i \int_0^\infty \frac{e^{izv}}{1 + v^4} dv$$

is the analytic continuation of  $f(z)$  to the region  $0 < \arg z < \pi$ .

- (d) (3 points) Discuss the complete asymptotic expansion for  $f(z)$  in  $0 < \arg z < \pi$ , noting the different behavior in different sectors.