

Homework Set 7 (Revised)

Read sections 10.1 and 10.2.

The WKB Approximation

1. (5 points) Find the leading-order asymptotic behavior of solutions to

$$y'' - (\log x)^2 y = 0. \quad (7.1)$$

as $x \rightarrow \infty$.

2. (9 points) Find the leading-order asymptotic behavior of solutions to

$$y'' = (x + x^\alpha)y, \quad (7.2)$$

as $x \rightarrow \infty$ and show that the leading-order behavior is exactly that of the Airy functions whenever $\alpha < -1/2$.

3. (13 points) Find the general solution of

$$w'' + \left[k^2 + \frac{1}{k(x^2 + 1)} \right] w = -k^2 \quad (7.3)$$

for large k . Construct enough terms so that your error is $o(k^{-3})$.

4. (13 points) Consider the system

$$x^2 y'' \sqrt{2} = [(x + 1)y + xz], \quad (7.4a)$$

$$x^2 z'' \sqrt{2} = -[xy + (x + 1)z], \quad (7.4b)$$

Show that

$$y(x) \sim \left(\frac{x^{5/8}}{\sqrt{2}} + \frac{x^{1/8}}{2} \right) e^{S_0(x)}$$
$$z(x) \sim \left(-\frac{x^{5/8}}{\sqrt{2}} + \frac{x^{1/8}}{2} \right) e^{S_0(x)}.$$

as $x \rightarrow \infty$, and determine $S_0(x)$. Why is it not appropriate to naïvely approximate the $x + 1$ terms in (7.4) by x ?