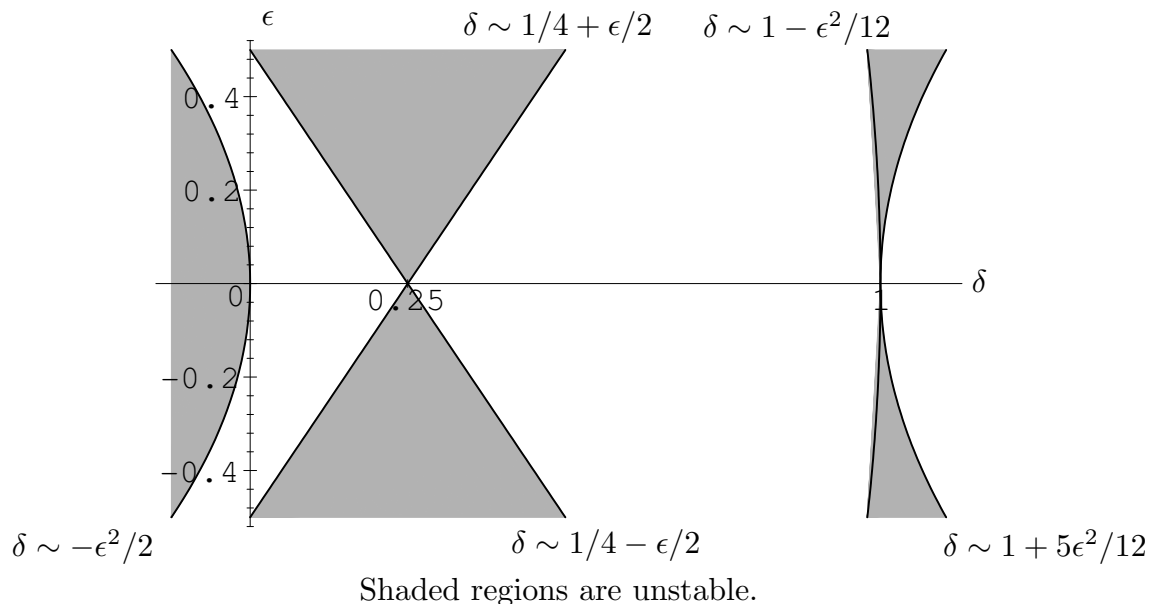


Homework Set 6

Read section 11.4.

Mathieu's Equation



1. Above is shown a graph of the (δ, ϵ) plane for the standard Mathieu equation. Now consider the Mathieu equation with an extra term added:

$$u'' + 2\beta u' + (\delta + \epsilon \cos t)u = 0, \quad |\epsilon| \ll 1, \quad (6.1)$$

where β is a constant. We wish to see how the stability diagram for $\beta = 0$ changes when $\beta \neq 0$.

- (a) (5 points) By introducing a new variable v , one can reduce the equation (6.1) for u to the standard Mathieu equation for v (perhaps with different parameters). Determine the relationship between u and v .
- (b) (9 points) Explain why if β is “large,” the solution will be dominated by the linear damping/growth term. Determine “large” for each of the special values $n^2/4$ of δ_0 .
- (c) (17 points) Determine the new stability curves for the cases $n = 0$, $n = 1$, and $n = 2$ when the linear and nonlinear effects balance [*i.e.*, when β is at the threshold values from part (b).]
- (d) (9 points) Sketch the new stability diagram.