

Updates

1. Writing assignment topics are due on Wednesday, March 10.
2. A makeup class will be held from 9:05–9:55 in GOR 320 on Friday, March 12.

Homework Set 4

Read section 9.5.

Singular BVPs

1. Consider the following singular boundary-value problem:

$$\epsilon y'' + \frac{y'}{x} - 2y = 0, \quad \epsilon^2 \leq x \leq 1, \quad 0 < \epsilon \ll 1, \quad (4.1a)$$

$$y(\epsilon^2) = 1, \quad y(1) = 1. \quad (4.1b)$$

- (a) (3 points) Construct any needed leading-order outer expansions.
- (b) (5 points) Construct any needed leading-order inner expansions.
- (c) (4 points) Construct and sketch the leading-order uniformly valid approximation of the solution.

Higher-Order ODEs

2. Consider the following system:

$$\epsilon v'' + u'v' = 0, \quad u(0) = u(1) = 0, \quad (4.2)$$

$$u'' = v, \quad v(0) = 2v_* > 0, \quad v(1) = 2\alpha^2 v_*, \quad \alpha > 1. \quad (4.3)$$

- (a) (6 points) Write down the proper scalings and equations for each needed layer. Explain why your choice of layers is unique. (*Hint: If proper placement of the layer is proving difficult, try examining the shape of the solution for any ϵ .*)
- (b) (4 points) Construct any necessary outer solutions and indicate the position of any layers.
- (c) (3 points) Sketch the solution.

3. Now we wish to examine the system (4.2) coupled to

$$u'' = -v, \quad v(0) = \bar{v} - v_*, \quad v(1) = \bar{v} + v_*, \quad \bar{v} > v_* > 0. \quad (4.4)$$

which replaces (4.3).

- (a) (4 points) Write down the proper scalings and identify the position for each needed layer. Explain why your choice of layers is unique. (*Hint: If proper placement of the layer is proving difficult, try examining the shape of the solution for any ϵ .*)
- (b) (3 points) Explain why $v'(0) = v'(1)$.
- (c) (5 points) Write down a leading-order uniformly valid solution for v and the leading-order outer solution for u .
- (d) (3 points) Sketch the solution.