

## Updates

Makeup classes will be held from 9:05–9:55 in GOR 320 on Friday, March 5 and Friday, March 12.

## Homework Set 3 (Revised)

Read sections 9.1–9.7.

### Linear ODEs

1. Consider the following equation:

$$\epsilon^4 y'' + x^2 y' + (x - \epsilon)y = 0, \quad y(0) = y(1) = 1. \quad (3.1)$$

- (a) (4 points) Construct any needed leading-order outer expansions.
- (b) (9 points) Construct any needed leading-order inner expansions.
- (c) (2 points) Construct the leading order uniform expansion.
- (d) (5 points) Using a computer algebra system, plot the error between the numerical solution of (3.1) and your answer to (c) for  $\epsilon = 0.2$ . Discuss the size of the error in various regions in light of what the size of the next-order term would be in each region.

### Nonlinear ODEs

2. (3 points) Show that in the model nonlinear example presented in class:

$$\epsilon y'' + yy' - y = 0, \quad y(0) = A, \quad y(1) = B,$$

if we make the substitutions

$$B \mapsto -A, \quad A \mapsto -B, \quad x \mapsto 1 - x, \quad y \mapsto -y,$$

we obtain the same differential equation as before.

This then shows how to extend our results to the region  $B < -A$ .

3. Consider the following problem:

$$\epsilon u'' - uu' - u = 0, \quad 0 \leq x \leq 1, \quad 0 < \epsilon \ll 1, \quad (3.2a)$$

$$u(0) - \alpha \epsilon u'(0) = 2, \quad u(1) = -2, \quad (3.2b)$$

where  $\alpha$  is a positive  $O(1)$  constant.

- (a) (5 points) Construct any outer expansions to the order needed to satisfy the boundary conditions.
- (b) (9 points) Construct any needed leading-order inner expansions.
- (c) (3 points) Construct and sketch the leading-order uniformly valid approximation of the solution(s).