

Homework Set 2

Read sections 7.1, 7.2, 9.1, and 9.2.

Regular Perturbations

1. We wish to track a projectile's motion. In this case we neglect the changing gravitational force but include the effects of air resistance. The resulting dimensional equation is

$$m \frac{d^2 \tilde{x}}{dt^2} + k \frac{d\tilde{x}}{dt} = -mg, \quad \tilde{x}(0) = 0, \quad \frac{d\tilde{x}}{dt}(0) = V. \quad (2.1)$$

- (a) (3 points) Using the appropriate scalings for a projectile near the Earth, obtain a dimensionless equation involving the parameter

$$\beta \equiv \frac{kV}{mg}.$$

Since the effects of air resistance are quite weak, we let $\beta \ll 1$.

- (b) (7 points) Using this assumption, show that the dimensionless time and height corresponding to the highest position of the projectile is given by

$$(t_{\max}, x_{\max}) = \left(1 - \frac{\beta}{2}, \frac{1}{2} - \frac{\beta}{3}\right) + o(\beta).$$

Singular Algebraic Equations

2. (10 points) Find the roots of

$$\epsilon x^3 + x^2 - 1 = 0, \quad 0 < \epsilon \ll 1$$

up to $O(\epsilon)$. (Throughout the course, this means find all terms that are not $o(\epsilon)$.)

Linear ODEs

3. Consider the following problem, similar to one I presented in class:

$$\begin{aligned} \epsilon y'' + y' + y &= 0, & 0 \leq x \leq 1, & \quad 0 < \epsilon \ll 1, \\ y'(0) &= A, & y(1) &= 1. \end{aligned}$$

- (a) (6 points) Calculate the outer expansion to $O(\epsilon)$.
- (b) (8 points) Calculate the inner expansion to $O(\epsilon)$.
- (c) (2 points) Calculate the uniform expansion to $O(\epsilon)$.
- (d) (4 points) Use a computer algebra system to plot $y(x)$ and $y'(x)$ when $A = 2$ and $\epsilon = 0.01$.