MATH 810-010 Asymptotics and Perturbation Methods
Prof. D. A. Edwards Due: May 19, 2010

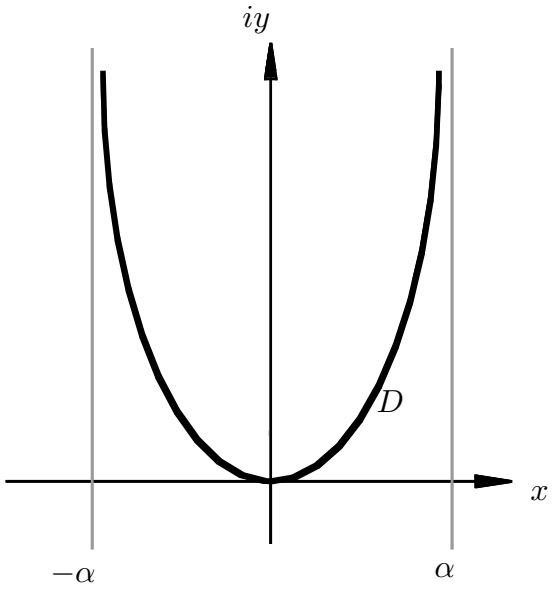
Updates

1. Writing assignments are due on Monday, May 10.

Homework Set 11

Read sections 6.6 and 6.7.

Saddle Points



1. Let

$$I(k) = \int_D e^{ik \sin^3 z}\, dz, \qquad k \in \mathcal{R}, \tag{11.1}$$

where D is the contour shown in the figure above, and the contour is traversed from left to right.

(a) (3 points) Show that if $\pi/6 < \alpha < \pi/2$, the integral is guaranteed to exist.
(b) (8 points) Show that

$$I(k) \sim \frac{\Gamma(1/3)}{k^{1/3}\sqrt{3}}, \qquad k \to \infty.$$

Asymptotic Expansions of Sums

2. (8 points) Show that

$$\sum_{k=0}^{n} \binom{n}{k} k! n^{-k} \sim \sqrt{\frac{\pi n}{2}} \text{ as } n \to \infty.$$

$$\textit{Hint: } k! n^{-k-1} = \int_0^\infty e^{-nx} x^k \, dx.$$

Partial Differential Equations

3. Consider the following equation and boundary conditions:

$$\epsilon u_{xx} = u_t + (\cos t) u_x, \qquad x > 0, \quad t > 0, \quad 0 < \epsilon \ll 1, \tag{11.2a}$$

$$u(x, 0) = 0, \qquad u(0, t) = 1. \tag{11.2b}$$

(a) (5 points) Construct the outer operator to the problem. Sketch the x-t plane, indicating any important subcharacteristics.
(b) (3 points) By considering the possibility of an initial layer, determine the outer solution.
(c) (13 points) Construct any needed inner solutions. (Do not worry about corner layers.)