Homework Set 10

Read section 6.6.

Steepest Descent

1. (15 points) page 314, exercise 6.81(a). You should find that

$$\int_0^1 e^{-xt^3}\,dt = (\sqrt{3}+i)\frac{\Gamma(1/3)}{6x^{1/3}} + e^{ix}\sum_{n=0}^{\infty}\frac{(2n)!}{n!(3ix)^{n+1}}.$$

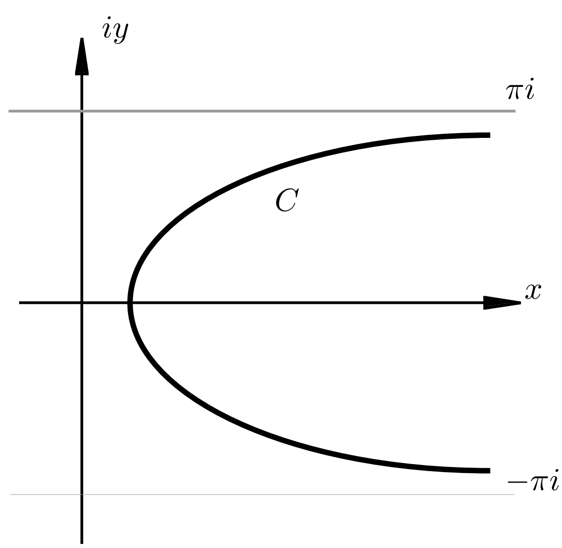
Saddle Points

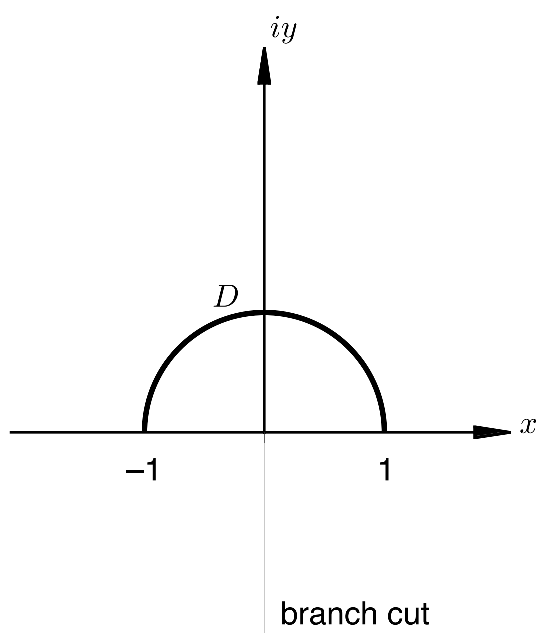
2. (10 points) The Bessel function of order ν may be represented as

$$J_\nu(x) = \frac{1}{2\pi i}\int_C e^{x\sinh t-\nu t}\,dt, \tag{10.1}$$

where C is the contour shown in the left figure below, and the contour is traversed counterclockwise. Use this representation to show Debye's result

$$J_\nu(\nu\operatorname{sech}\alpha) \sim \frac{e^{\nu(\tanh\alpha-\alpha)}}{\sqrt{2\pi\nu\tanh\alpha}} \text{ as } \nu\to\infty.$$





3. (15 points) Consider the function

$$I(z) = \int_D e^{z(s-\log s)}\, ds,$$

where D is the contour shown on the previous page (traversed counterclockwise) and we have chosen the branch where $\log 1 = 0$. Find the first term in the asymptotic expansion of $I(z)$ as $z \to \infty$ for all $0 \le \arg z < 2\pi$.