Homework Set 1

Read sections 6.3, 7.1.

Order Notation

1. (3 points) Verify example 1 on page 318.
2. Verify the following equalities. α and β be are positive $O(1)$ constants and all calculations should be made in the limit that $\epsilon \to 0^+$.
   (a) (2 points)
   $$\epsilon = o\left(\epsilon|\log\epsilon|^{\beta}\right).$$
   (b) (2 points)
   $$\epsilon|\log\epsilon|^{\beta} = o\left(\epsilon^{1-\alpha}\right).$$
   This "nesting" of logarithmic terms makes using asymptotic expansions involving logarithms an extremely delicate undertaking.
   (c) (2 points)
   $$e^{-\alpha/\epsilon} = o\left(\epsilon^{\beta}\right).$$
   This last equality will become crucial in the following few weeks.
3. Let $f(x)$ and $g(x)$ be functions at least as large as $O(1)$. (In other words, let $[f(x)]^{-1}$ and $[g(x)]^{-1}$ be $O(1)$.)
   (a) (5 points) Show by counterexample that $f(x) \sim g(x)$ does not necessarily imply that
   $$e^{f(x)} \sim e^{g(x)}. \tag{1.1}$$
   (b) (3 points) Show that (1.1) does indeed hold if $f(x) = g(x) + o(1)$.

Asymptotic Expansions

4. (5 points) page 361, exercise 7.2. Calculate the first three nonzero terms in the expansion.
5. (7 points) page 361, exercise 7.3

6. Consider the function

$$\operatorname{erfc} z = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2}\, dt.$$

(a) (9 points) Using repeated integration by parts, show that

$$\operatorname{erfc} z \sim \frac{e^{-z^2}}{z\sqrt{\pi}} \left[1 + \sum_{n=1}^{\infty} (-1)^n \frac{(1)(3)\cdots(2n-1)}{(2z^2)^n} \right], \qquad z \to \infty. \tag{1.2}$$

(b) (2 points) Using the ratio test outlined in class, estimate the optimal number of terms in the expansion for $z = 2$, $z = 3$, and $z = 4$.