

Writing Assignment

As advanced students in the sciences, it is reasonable to assume that at some point in your career you will need to communicate mathematical reasoning in writing to someone, whether it be for a paper in a published journal, a Ph.D. dissertation or master's thesis, or a "white paper" for one's employer. Though each homework assignment in and of itself is a written assignment where you (should) explain your mathematical reasoning in writing, it is much less structured than a standard paper. In addition, your audience is one which is very familiar with the material at hand. Therefore, you can be justified in skipping some steps in your analysis.

In order to give you some practice in scientific writing, I would like you to write up *thoroughly* your solution (using perturbation methods) of an applied problem of your own choosing. In theory, this would be a problem closely related to your area of mathematical interest. When working on the problem, you should try to keep in mind the techniques we have learned in this course. How should the governing equations be scaled? What effects are important/unimportant? How will a perturbation analysis simplify the problem? I want you to gain practice in expressing your ideas, rather than testing your knowledge of difficult mathematical concepts.

The assignment consists of four parts:

1. On March 4 you should submit a brief description of the topic you wish to explore in your paper. The description should be similar to an abstract: short, but indicative of the problem at hand and the mathematical techniques you think might be required. That way, I may be able to assist you in finding references and avoiding mathematical pitfalls which would needlessly complicate the project.
2. On March 29 you should submit an outline of your project. It should consist of
 - (a) a short introduction to the problem,
 - (b) a description or derivation of the equations and a nondimensionalization thereof, and
 - (c) a brief description of how your perturbation analysis will proceed.At this time I will perform another check just to make sure you aren't trying to do a thesis-level problem in a few weeks.
3. On May 10, you should submit the written paper, which should consist of
 - (a) an introduction to the physical problem.
 - (b) your model and any simplifications you employed.
 - (c) a careful exposition of how you solved the problem. This should be written for an audience which has some knowledge of the mathematical and background material, but not an extensive one. Pretend you are teaching the class.
 - (d) an analysis of your mathematical results, and what they reveal about the real-world problem. Were the final equations too simple to capture the real-world behavior? Does your solution have any useful predictive capabilities?Though it would be nice if the paper were typed, it is not required.
4. Once the papers have been turned in and I have had a chance to examine them, we will discuss them, with each of you sharing the problem on which you worked, how you attacked

it mathematically, and what you learned from the experience. This discussion is **NOT** to be a formal presentation.

Sample Topics

1. **BIOLOGY.** Consider a system of interrelated species. Postulate the dependence of their population growth rates as a function of their populations and other conditions. Deduce steady states for the system, and discuss your results in the limit of small (*i.e.*, endangered) populations.
2. **ENVIRONMENTAL ENGINEERING.**
 - (a) Radioactive waste has been placed in a storage facility near groundwater aquifers. Estimate the needed thickness of the facility walls in order to maintain the safety of the water supply.
 - (b) Thermal waste is ejected from a factory into a river. However, raising the temperature too much will kill fish living in the river. Optimize the placement of vents to minimize wildlife deaths.
3. **ASTRONOMY.** Model a multiple-body planetary system. What sort of catastrophic phenomena can cause a planet to depart from its orbit? Are there situations where a planet can leave its orbit on a long time scale?
4. **ACTUARIAL SCIENCE.** An insurance company knows its own tolerance for risk, but has various models from which to choose in order to model its customers' tolerance for risk. Formulate a model which will indicate the most cost-effective way for the insurers to set premiums.
5. **POLITICAL SCIENCE.** Present alternatives to the electoral college. Indicate how each might optimize the voters' choices in a way not currently done in the political system. Then indicate how candidates could thwart each of the systems.
6. **RISK MANAGEMENT.**
 - (a) A propane gas leak occurs in a shallow valley. Model the system, paying particular attention to the flammability limit and its sensitivity on wind speed.
 - (b) A credit card company wishes to identify those customers who are at risk for delinquent payments. Propose a model to do so.
7. **INDUSTRIAL DESIGN.**
 - (a) Design an efficient filter which is also cost-effective to replace, either through reverse-flow mechanisms or other devices.
 - (b) Calculate the optimal speed at which thin sheets of newsprint can be effectively run through a printing press without wrinkling or tearing.
8. **SPORTS.** Assume that Major League Baseball had a draft of the same sort as other professional sports associations; that is, that the worst teams in baseball would get the top draft picks. However, win-loss records alone are misleading since teams play more often within their division than without and within their league than without (starting next season). Devise an alternative way of choosing the "worst" teams.
9. **PURSUIT.** Four soldiers set out to pursue an enemy soldier on foot. What is their optimal

strategy to capture the soldier before he reaches an enemy stronghold? What is the enemy's optimal strategy?

10. **FINANCE.** A single financial option can now be based on a wide variety of stocks. How does one simplify the multiple levels of variability created by the different stocks in order to value the option effectively?