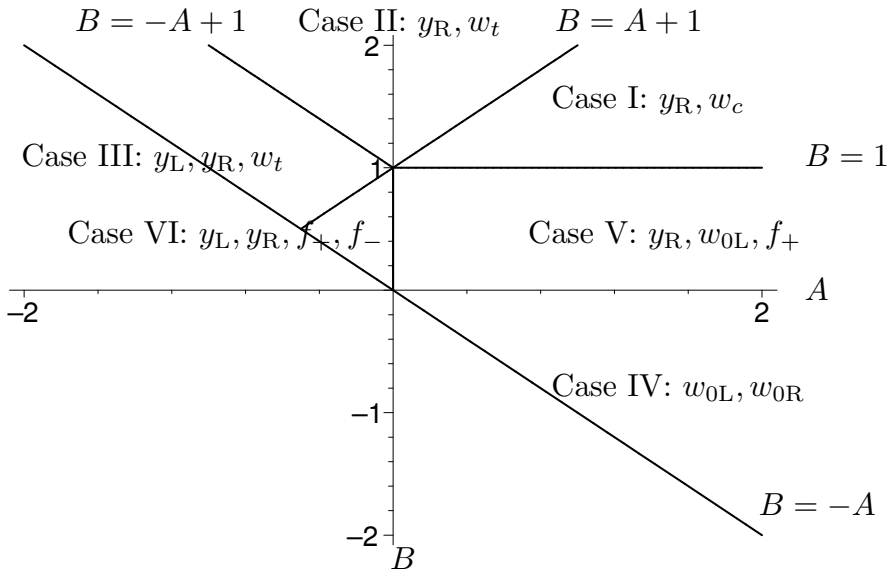
# Nonlinear Example (Revised)

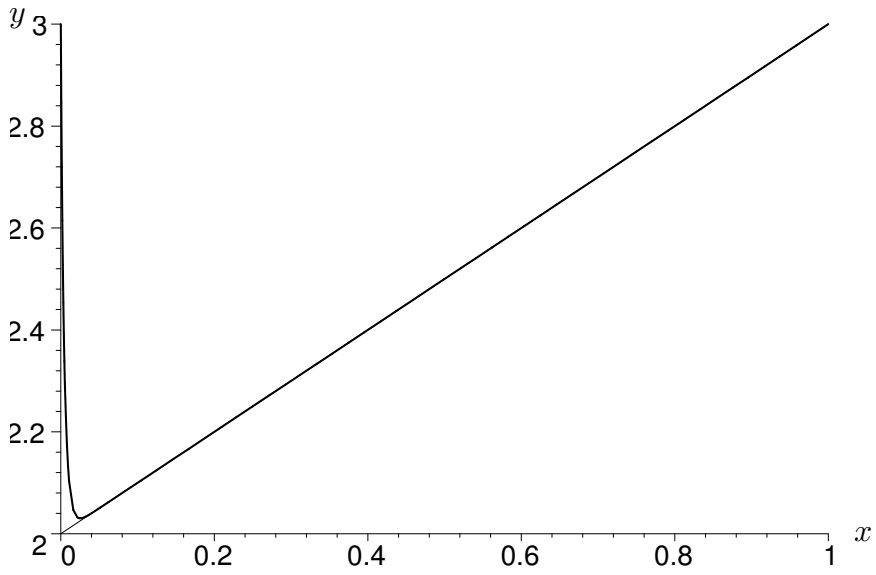
Consider the following system:

$$\epsilon y'' + yy' - y = 0, \qquad y(0) = A, \quad y(1) = B.$$

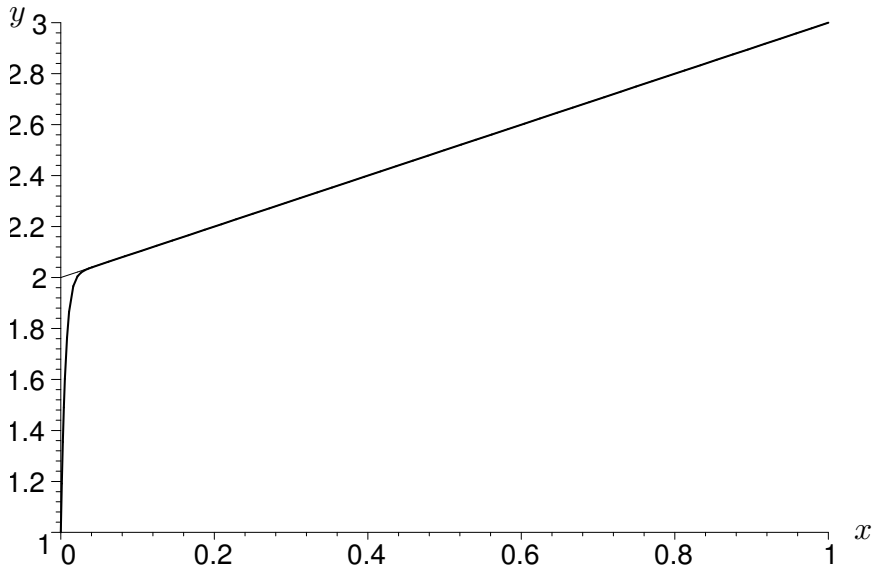
The behavior of the solution this system depends on the values of $A$ and $B$. Here is how the $A$-$B$ parameter plane divides up for $A > -B$:



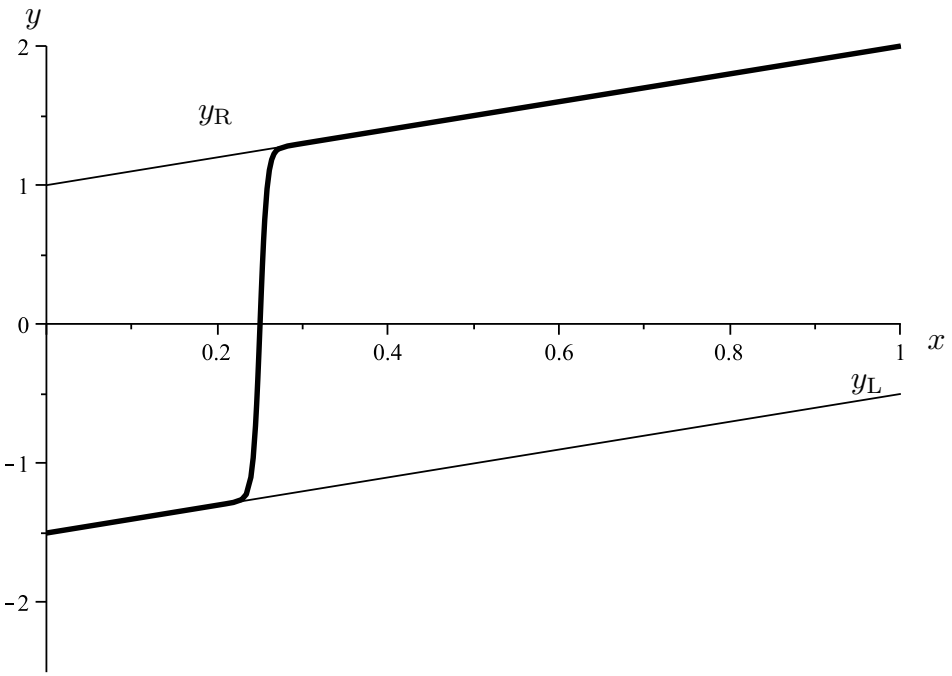

$A$-$B$ parameter plane



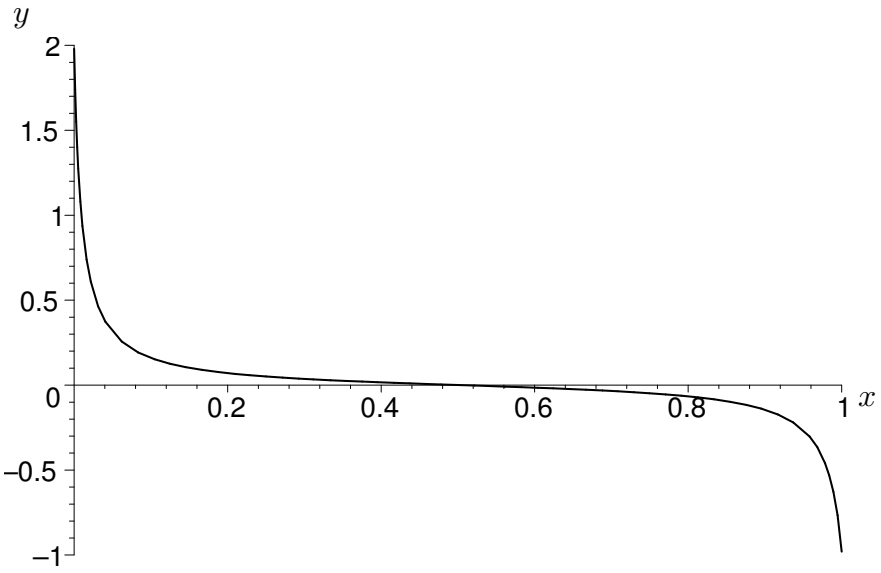

Case I (boundary coth layer): $A = B = 3$, $\epsilon = 0.01$.
Thin line: outer solution. Thick line: uniform solution.




Case II (boundary tanh layer): $A = 1$, $B = 3$, $\epsilon = 0.01$.
Thin line: outer solution. Thick line: uniform solution.




Case III (internal tanh layer): $A = -3/2$, $B = 2$, $\epsilon = 0.01$.
Thin lines: outer solutions. Thick line: uniform solution.

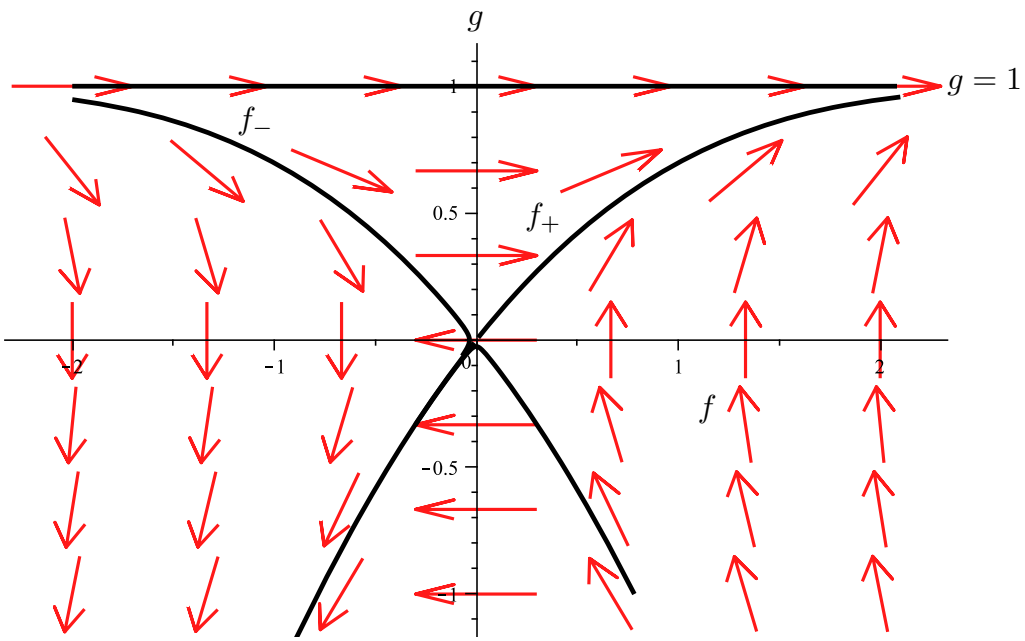



Case IV (two algebraic boundary layers): $A = 2$, $B = -1$, $\epsilon = 0.01$.

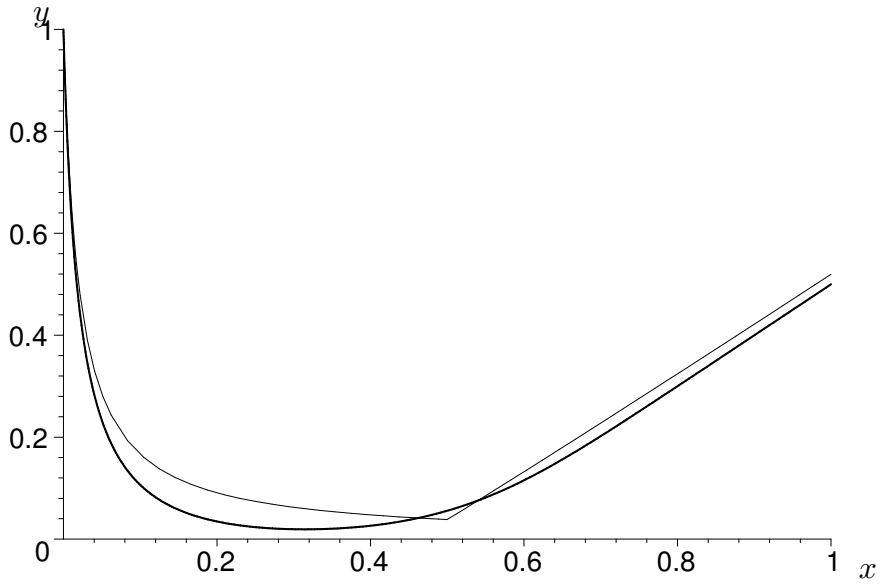
The system for the internal layer is given by

$$\begin{aligned} \frac{df}{d\zeta} &= g, \\ \frac{dg}{d\zeta} &= f(1-g). \end{aligned} \qquad g(-\infty) = 0, \quad g(\infty) = 1.$$

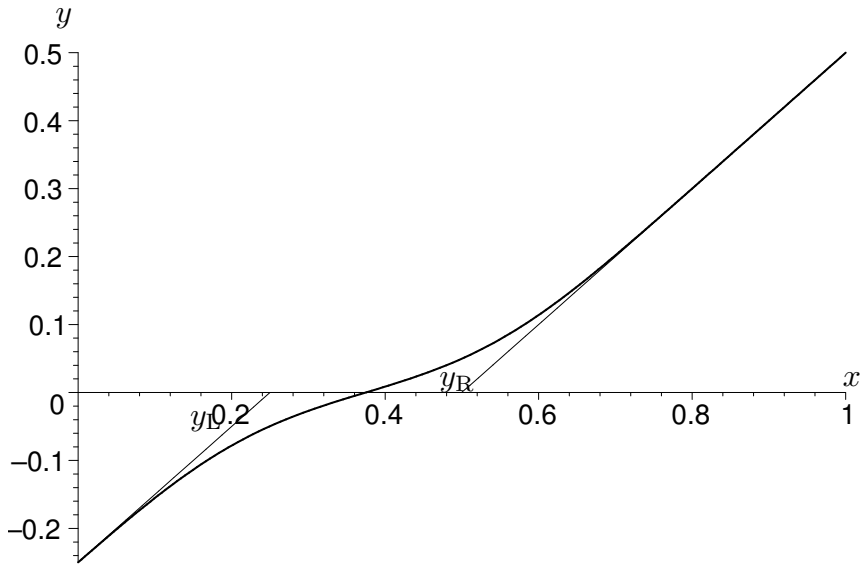
Here is the phase plane.




Phase plane for $f(\zeta)$.




Case V (algebraic boundary and corner layer): $A = 1$, $B = 1/2$, $\epsilon = 0.01$.
Thin line: solution without corner layer. Thick line: numerical solution.




Case VI (two corner layers): $A = -1/4$, $B = 1/2$, $\epsilon = 0.01$.
Thin line: outer solutions. Thick line: numerical solution.