MATH 810 Asymptotics and Perturbation Methods
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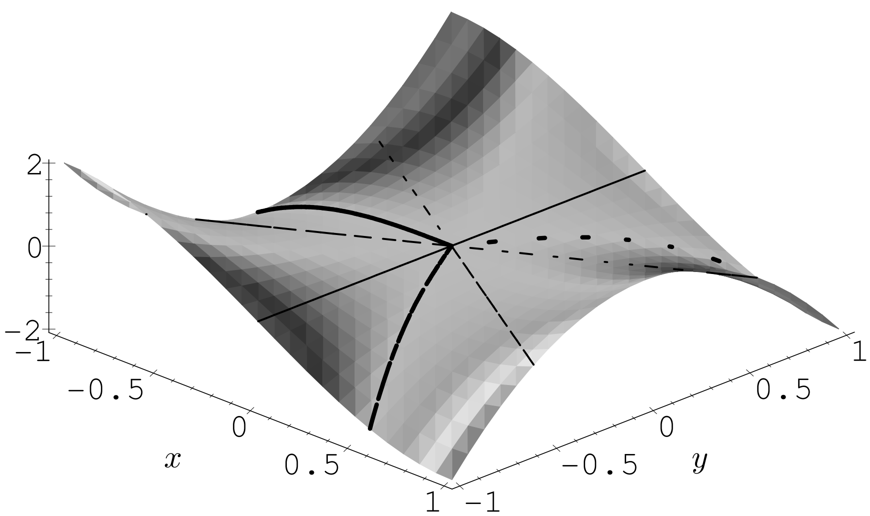
Monkey Saddle

Consider the following integral, which is an idealization of the approximation to a general complex integral about a saddle point:

$$I = \int_C f(z) e^{z^3}\, dz,$$

where C is an arbitrary contour. A graph of the real part of the exponent z^3 is shown. The shading function is $\Im(z^3)$.



Let $z = x + iy$. Then

$$\Re(z^3) = x^3 - 3xy^2, \tag{1}$$

$$\Im(z^2) = 3x^2 y - y^3. \tag{2}$$

The thin lines shown are the lines $x = 0$, $y = \pm x/\sqrt{3}$, along which $\Re(z^3)$ is constant. Note that along these lines the phase changes drastically. The thick curves have $y = 0$, $y = \pm x\sqrt{3}$. We note that these are the curves of steepest descent, and that the phase is constant along each curve.