MATH 672-010 — Vector Spaces
Prof. D. A. Edwards — Due: Oct. 20, 2008

Homework Set 6

Read pp. 80, 81, 87–93.

Polynomials, Diagonal Matrices

1. (5 points) page 95, number 14
2. (8 points) Two operators S and $T \in \mathcal{L}(V)$ are said to be *simultaneously diagonalizable* if $\mathcal{M}(T, Z, Z)$ and $\mathcal{M}(S, Z, Z)$ are diagonal for the same basis Z of V. Prove that if S and T are simultaneously diagonalizable, $ST = TS$.
3. (5 points) Let $T \in \mathcal{L}(V)$ be an invertible operator such that $\mathcal{M}(T, Z, Z)$ is diagonal for some basis Z of V. Find a basis B of V such that $\mathcal{M}(T^{-1}, B, B)$ is also diagonal. If the diagonal entries of $\mathcal{M}(T, Z, Z)$ are given by t_{ii}, find the diagonal entries of $\mathcal{M}(T^{-1}, B, B)$.
4. (8 points) Let $T \in \mathcal{L}(V)$, $\dim V = n$, λ be an eigenvalue of T. Furthermore, assume that the dimension of the eigenspace of T corresponding to λ is m. Show that there exists a basis W for V such that

$$\mathcal{M}(T, W, W) = \begin{pmatrix} \lambda I & B \\ O & C \end{pmatrix}, \quad B \in \mathcal{R}^{m \times (n-m)},\ C \in \mathcal{R}^{(n-m) \times (n-m)},$$

where I is the identity matrix in $\mathcal{R}^{m \times m}$, and O is the zero matrix in $\mathcal{R}^{(n-m) \times m}$.

Existence of Eigenvalues

5. (10 points) page 95, number 16. Rather than simply providing a counterexample, prove as strong a statement as possible.
6. (4 points) page 96, number 24