

Homework Set 1

Read chapter 1.

Vector Spaces

- (2 points) Prove that the multiplicative identity for a vector space is unique.
- (4 points per part) For the following sets, either prove the set is a vector space or find a property which the set violates.
 - Vectors: solutions to the differential equation $y'' + y = 0$
Vector addition: $f(t) + g(t) = (f + g)(t)$
Scalar multiplication: $c[f(t)] = (cf)(t)$
 - Vectors in \mathcal{C}^2
Vector addition: $(x_1, x_2) + (y_1, y_2) = (x_1 + 3y_1, x_2 + 2y_2)$
Scalar multiplication: $c(x_1, x_2) = (cx_1, cx_2)$

Subspaces

- (5 points) Prove that the only subspaces of \mathcal{R}^2 are $\{\mathbf{0}\}$, \mathcal{R}^2 , and subspaces of the form $W_{\mathbf{v}} = \{c\mathbf{v}, c \in \mathcal{R}\}$ for some $\mathbf{v} \in \mathcal{R}^2$.
- (2 points per part) Consider the vector space $\mathcal{C}(\mathcal{R})$ (the space of all real-valued continuous functions defined for all \mathcal{R}). For each of the following subsets, either prove the set is a subspace of $\mathcal{C}(\mathcal{R})$ or find a property which the set violates.
 - All decreasing functions.
 - All differentiable functions.
 - All integrable functions.

Direct Sums

- Let M_+ be the set of all symmetric matrices in $\mathcal{R}^{n \times n}$, and M_- be the set of all antisymmetric matrices in $\mathcal{R}^{n \times n}$.
 - (3 points) Verify that M_- is a subspace of $\mathcal{R}^{n \times n}$.
 - (6 points) Show that $\mathcal{R}^{n \times n} = M_+ \oplus M_-$.
- Let $\mathcal{C}_e(\mathcal{R})$ be the set of all even continuous functions defined on \mathcal{R} , and let $\mathcal{C}_o(\mathcal{R})$ be the set of all odd continuous functions defined on \mathcal{R} .
 - (4 points) Prove that $\mathcal{C}_e(\mathcal{R})$ and $\mathcal{C}_o(\mathcal{R})$ are subspaces of $\mathcal{C}(\mathcal{R})$.
 - (6 points) Using the *definition* of the direct sum, prove that $\mathcal{C}(\mathcal{R}) = \mathcal{C}_e(\mathcal{R}) \oplus \mathcal{C}_o(\mathcal{R})$.