Homework Set 5

Read sections 3.3, 3.4, 3.9, 6.2.

The Greeks

1. (a) (5 points) Calculate \( \Delta \) for a digital call.
   (b) (4 points) Suppose that \( t \) is near expiry and \( S \) is near the strike. In particular, let \( t = T - \epsilon^2, S = K(1 + k\epsilon) \), where \( 0 < \epsilon \ll 1 \). What happens to \( \Delta \) in this instance? Discuss the practicality of hedging such an option in this case.

Put-Call Parity

2. (8 points) Show mathematically that the vega for a European put is the same as for a European call. Explain your result from a portfolio perspective.
Other Payoffs

3. The payoff diagram shown is for a *condor* spread. Denote its option value by \( V_z \).
   (a) (6 points) Construct a portfolio \( \Pi_B \) of two bullish spreads \( B(t; K_b, K_s) \) that replicates this payoff. Draw the payoff diagrams of the individual spreads.
   Write \( V_z \) in terms of the values of various call options (you needn’t write out the full Black-Scholes formulas).
   (b) (9 points) Show that the sensitivity of \( V_z \) to changes in \( z \) is given by
   \[
e^{-r(T-t)}[N(d_2^+) - N(d_2^-)], \quad d_2^\pm = \frac{\log(S/(K \pm z)) + (r - \sigma^2/2)(T - t)}{\sigma \sqrt{T - t}}.
   \]
   (c) (2 points) Use your answer to (b) to show that a condor is always cheaper than a butterfly for the same values of \( K_\pm \).
   (d) (2 points) Explain your answer to (c) financially.

Dividends

4. (4 points) page 104, exercise 1