Homework Set 1

Read chapter 1, sections 3.1, 3.2, 6.3.

Forwards and Futures

1. (9 points) Another common future is a currency future, where you enter into a contract (paid for in currency 1) to obtain one unit of currency 2 at some point \( T \) in the future. Suppose that the exchange rate today between two currencies is \( S(0) \), and that the risk-free rate for currency \( j \) is \( r_j \). Show that the currency forward price is given by

\[
F(0; T) = S(0)e^{(r_1 - r_2)T}.
\]  

(1.1)

Make sure to explain your reasoning in terms of both the buyer and the seller.

2. A prepaid forward contract is one where the buyer has to pay the seller at the time the contract is written, rather than at the settlement date.

(a) (4 points) Calculate the price \( F(0; 0) \) of a prepaid forward contract in the case of no dividends. Make sure to explain your reasoning in terms of both the buyer and the seller.

(b) (9 points) Suppose that a stock pays a fixed cash dividend amount \( D(t_i) \) at fixed times \( t_i \), where \( 0 < t_1 < t_2 < \cdots < t_n < T \). Show that in this case, the prepaid forward contract price is given by

\[
F(0; 0) = S(0) - \sum_{i=1}^{n} D(t_i)e^{-r t_i}.
\]

Verify that it reduces to your answer to (a) if \( D(t_i) = 0 \).
Hedging

3. Let’s generalize our gambling example. Suppose that I have a $100 ticket that will pay a profit of $P_1$ if team 1 wins. Moreover, suppose that the current odds for team 2 winning are $P_2:1$.

(a) (5 points) Find the amount $x$ I should bet on team 2 to hedge my bet.

(b) (7 points) Find my expected profit $E$ in this case. Show that $E$ is an increasing function of $P_2$, bounded between the winning and losing sides of the $100 bet. Interpret the limits that $P_2 \to 0$ and $P_2 \to \infty$.

(c) (4 points) Under what conditions will your expected profit will be 0? Explain your answer in terms of your bet $x$.

(d) (2 points) Use your formulas to verify the example given in class.