Review Material (4/6 Revision)

In this class we will be assuming various material from previous courses that you may not have used for a while. If you are rusty on any of those topics, I suggest reviewing your notes or textbook for the class.

MATH 242

Suppose that we have a function \( f(x) \) that we know everything about at a point \( x = a \). If we want to estimate the function in a neighborhood (usually small) about \( x = a \), we use the Taylor series, which is normally written this way:

\[
f(a + h) = f(a) + hf'(a) + \frac{h^2}{2} f''(a) + O(h^3),
\]

where “\( O \)” means “proportional to.”

MATH 302

To solve the homogeneous ODE with constant coefficients

\[
ay'' + by' + cy = 0,
\]

we try a solution of the form \( y = e^{\lambda x} \). This will yield the characteristic equation

\[
a\lambda^2 + b\lambda + c = 0.
\]

Solving this equation will yield two constants \( \lambda \) which correspond to the two linearly independent solutions of (1).

Next, for an Euler equation

\[
at^2 \ddot{y} + bt\dot{y} + cy = 0,
\]

substituting \( y = t^\lambda \) again yields a quadratic equation for \( \lambda \):

\[
a\lambda(\lambda - 1) + b\lambda + c = 0.
\]

Solving this equation will yield two constants \( \lambda \) which correspond to the two linearly independent solutions of (2).

The Laplace transform \( \hat{f} \equiv \mathcal{L}[f] \) allows us to simplify PDEs by turning them into ODEs. For our purposes, one must remember that

\[
\mathcal{L}\left[\frac{df}{dt}\right] = s\hat{f} - f(0).
\]
Then one can solve the resulting equation for $\hat{f}$ and use tables to find the corresponding form for $f$.

The **delta function** $\delta(x)$ has the following properties:

$$
\delta(x) = \begin{cases} 
0, & x \neq 0, \\
\infty, & x = 0,
\end{cases}
\int_a^b \delta(x) \, dx = 1, \quad a < 0 < b.
$$

**MATH 350**

Recall that the mean of any random variable $X$ is given by $E[X]$, and the variance is given by $E[(X - E[X])^2] = E[X^2] - (E[X])^2$.

We say that a random variable $X$ is *normally distributed* with mean $\mu$ and variance $\sigma^2$ if its probability density function $n(x)$ is given by

$$
n(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2}\right).
$$

The notation for this is $X \sim \mathcal{N}(\mu, \sigma)$. We will use the notation $N(x)$ to denote the *cumulative* normal density function:

$$
N(x) = \int_{-\infty}^x n(t) \, dt.
$$

Note that since $n(x)$ is symmetric about $x = 0$, $N(-x) = 1 - N(x)$.

**Finance**

If we invest $1 at a rate $r$, compounded continuously, at time $t$ we will have $e^{rt}$. Similarly, if we *want* $1 at time $t$, we need invest only $e^{-rt}$ now. $e^{-rt}$ is called the *present* or *discounted* value of 1 at time $t$. 