The Greeks

As discussed in class, the option price for a European call is given by

\[
V(S, \tau) = SN(d_1) - Ke^{-r\tau}N(d_2),
\]

where \( N(\cdot) \) is the cumulative normal probability density function and

\[
d_1 = \frac{\log(S/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = \frac{\log(S/K) + (r - \sigma^2/2)\tau}{\sigma\sqrt{\tau}}.
\]

Now we wish to examine the sensitivity of the option prices to various parameters, \textit{i.e.}, the \textit{Greeks}. Note the volatility in these graphs (0.2) is smaller than in the graphs of the call, which is 0.5.

Graph of \( \Delta \) vs. \( S \) for the European call with \( K = 3, \sigma = 0.2, T = 1, r = 0.05 \).

In increasing order of thickness: \( \tau = 0, 1/3, 2/3, 1 \).

As shown in the above diagram, we see that as \( \tau \) decreases (\( t \) increases), \( \Delta \) sharpens until it becomes the Heaviside function, which describes whether the share is needed or not at expiry.
Graph of $\Gamma$ vs. $S$ for the European call with $K = 3$, $\sigma = 0.2$, $T = 1$, $r = 0.05$.
In increasing order of thickness: $\tau = 0, 1/3, 2/3, 1$.

Graph of Vega vs. $S$ for the European call with $K = 3$, $\sigma = 0.2$, $T = 1$, $r = 0.05$.
In increasing order of thickness: $\tau = 0, 1/3, 2/3, 1$. 
Graph of $\theta$ vs. $S$ for the European call with $K = 3$, $\sigma = 0.2$, $T = 1$, $r = 0.05$.
In increasing order of thickness: $\tau = 0, 1/3, 2/3, 1$.

Graph of $\rho$ vs. $S$ for the European call with $K = 3$, $\sigma = 0.2$, $T = 1$, $r = 0.05$.
In increasing order of thickness: $\tau = 0, 1/3, 2/3, 1$. 