The European Call (Revised)

As discussed in class, the option price for a European call is given by

\[ V(S, \tau) = SN(d_1) - Ke^{-r\tau}N(d_2), \]  

(1)

where

\[ N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u^2/2} \, du, \]

\[ d_1 = \frac{\log(S/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \]

\[ d_2 = \frac{\log(S/K) + (r - \sigma^2/2)\tau}{\sigma\sqrt{\tau}}. \]

Graph of (1) vs. \( S \) for \( K = 3, \sigma = 0.5, T = 1, r = 0.05 \). In increasing order of thickness: \( \tau = 0, 1/3, 2/3, 1 \).

As shown in the above diagram, we see that as \( \tau \) decreases (\( t \) increases), the option price sharpens until it becomes the payoff function.
Graph of (1) vs. $K$ for $S = 3$, $\sigma = 0.5$, $T = 1$, $r = 0.05$. In increasing order of thickness: $\tau = 0, 1/3, 2/3, 1$.

Typically the price is quoted for various strikes given an asset price today, as shown in the figure above.