Continuous Dividends

We showed in class that the value of a European call with continuous dividends is given by

\[
V(S, \tau) = [e^{-\delta(T-t)}S]N(d_1) - [Ke^{-r\tau}]N(d_2),
\]

where

\[
d_{1,2} = \frac{\log(S/K) + (r - \delta \pm \sigma^2/2)\tau}{\sigma\sqrt{\tau}}.
\]

Here you can think of replacing \( r \) in \( d_{1,2} \) by \( r - \delta \), or \( S \) by \( s \). Note that (1) reduces to our previous expression when \( \delta = 0 \).

Graph of \( V \) vs. \( S \) for \( K = 3, \sigma = 0.2, T = 1, r = 0.05 \). Dashed line: payoff function. Other lines have \( t = 0 \). In decreasing order of thickness: \( \delta = 0, 0.04, 0.08 \).

This diagram shows the value of a European call option when written and at expiry for different dividends. Note that when no dividends are paid, the value of the option is always more than the payoff, but that changes as the dividend rate increases.