Compounds and Choosers (Revised)

Let $C_c(t; K_c, T_c)$ be the value of a compound call-on-call option with strike $K_c$ and expiration $T_c$. Then its payoff is given by

$$C_c(S, T_c) = [C_u(T_u; K_u, T_u) - K_c]^+,\]$$

where $C_u(t; K_u, T_u)$ is the value at time $t$ of the underlying call option (which has strike $K_u$ and expiration $T_u > T_c$). The payoff diagram is shown below, which has the following conventions:

- The dotted line shows the value $C_u$ of the underlying option at its expiration $T_u$. (Values are measured on the left scale.)
- The dashed curve shows the value $C_u$ of the underlying option at the expiration $T_c$ of the compound option. Note that it is curved, similar to those on the European call handout for time before expiry. (Values are measured on the left scale.)
- The solid curve shows the value $C_c$ of the compound option at its expiration $T_c$. Here values are measured on the right scale. Note that the option is worthless for values of the underlying option below the strike.

![Fig. 1. Plot for $K_u = 3$, $T_u = 1$, $\sigma = 0.5$, $T_c = 1/3$, $r = 0.05$, $K_c = 2/3$.](image)

Note that the payoff for $C_c$ is curved, which makes analytical expressions more difficult to calculate.
Let $C_{ch}(t; K_{ch}, T_{ch})$ be the value of a chooser option with strike $K_{ch}$ and expiration $T_{ch}$. Then its payoff is given by

$$C_{ch}(S, T_{ch}) = \max\{C_u(T_{ch}; K_u, T_u) - K_{ch}, P_u(T_{ch}; K_u, T_u) - K_{ch}, 0\},$$

where $C_u$ is as defined above, and $P_u(t; K_u, T_u)$ is the value of the corresponding put option. The payoff diagram is shown below, which has the following conventions:

- The dotted line and dashed curve are as in Fig. 1, except for the put rather than the call.
- The solid curve shows the value $C_{ch}$ of the chooser option at its expiration $T_{ch}$. Here values are measured on the right scale. The right portion of the curve results from exercising the call, and is hence the same as in Fig. 1. Note also that there is a region where both the underlying call and underlying put are below the strike, and the chooser is worthless.

![Plot](image_url)

**Fig. 2.** Plot for $K_u = 3$, $T_u = 1$, $\sigma = 0.5$, $T_{ch} = 1/3$, $r = 0.05$, $K_{ch} = 2/3$. 