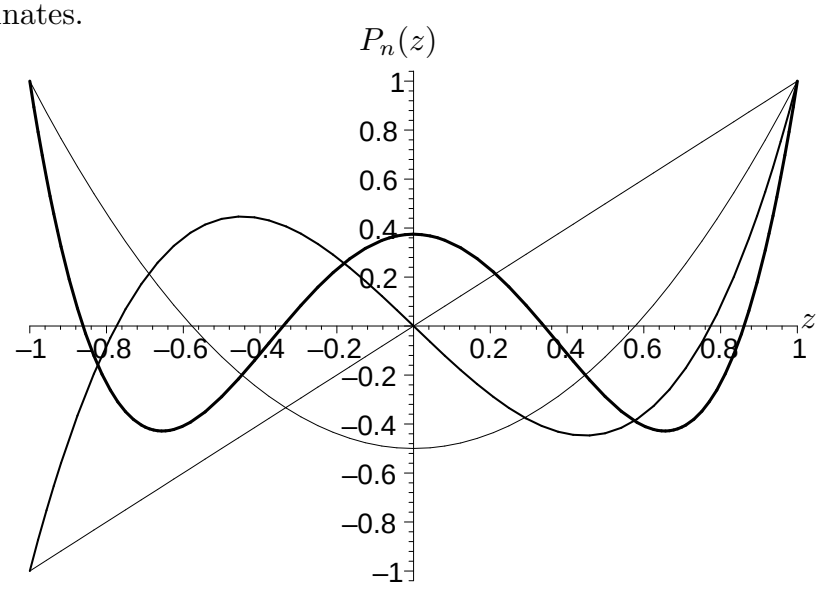
Legendre Polynomials

If n is an integer, then there exists a polynomial solution to the Legendre equation of order n:

$$(1-z^2)\frac{d^2y}{dz^2} - 2z\frac{dy}{dz} + n(n+1)y = 0, \qquad z \in [-1,1].$$

This solution is called the *Legendre polynomial* $P_n(z)$. z is taken in this range because most often, $z = \cos\phi$, where ϕ is the *colatitude* (angle measured from the north pole) in spherical coordinates.



$P_n(z)$ *vs.* z for $n = 1, 2, 3, 4$ (in increasing order of thickness).

Here is a plot of $P_n(z)$ for various n. Note that:

1. P_n is odd if n is odd and even if n is even.
2. $P_n(1) = 1$.
3. $|P_n(z)| \le 1$.
4. The zeroes *interlace*; that is,

$$p_{n-1,k-1} < p_{n,k} < p_{n-1,k} \text{ for all } i,k.$$