The Elastica

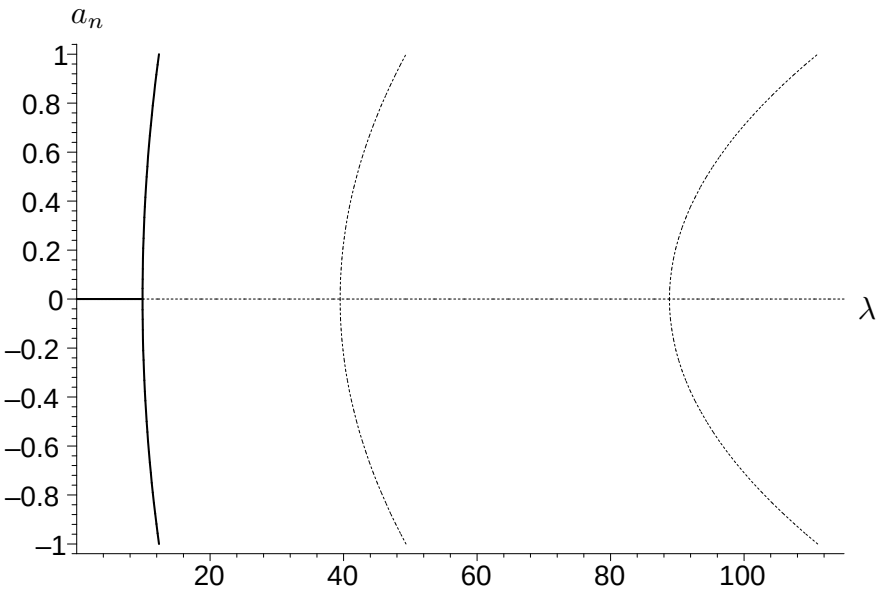
In class we found that the eigenfunctions of the elastica equation:

$$y'' + \left[\lambda - \frac{1}{2}\int_0^1 (y')^2\,dx\right] = 0, \qquad y(0) = y(1) = 0, \tag{1}$$

were given by

$$y_n = a_n \sin(n\pi x), \qquad \frac{a_n^2}{4} = \frac{\lambda}{n^2\pi^2} - 1, \quad n \geq 1,$$
$$y_0 = 0.$$

A stability diagram is shown below. Even though our stability analysis was **NOT** correct for the elastica, it did yield the correct classification of the eigenfunctions.



Eigenfunctions of (1). Stable states are thick solid lines.