Traveling Waves

Consider the Korteweg-de Vries equation

\[
\frac{\partial \eta}{\partial t} + \eta \frac{\partial \eta}{\partial x} + \frac{\partial^3 \eta}{\partial x^3} = 0, \quad -\infty < x < \infty, \quad \eta(x, t) \to 0 \text{ as } |x| \to \infty.
\]

In class we found a traveling-wave solution to be

\[
\eta(x, t) = 3V \text{ sech}^2 \left( \frac{\sqrt{V}(x - Vt)}{2} \right). \tag{1}
\]

Graphs of the solution are shown below. Note that waves with larger amplitude are narrower and travel faster.

Graph of (1) for \(V = 1\) (solid) and \(V = 3\) (dashed).
In increasing order of thickness: \(t = 0, 1, 2, 3\).