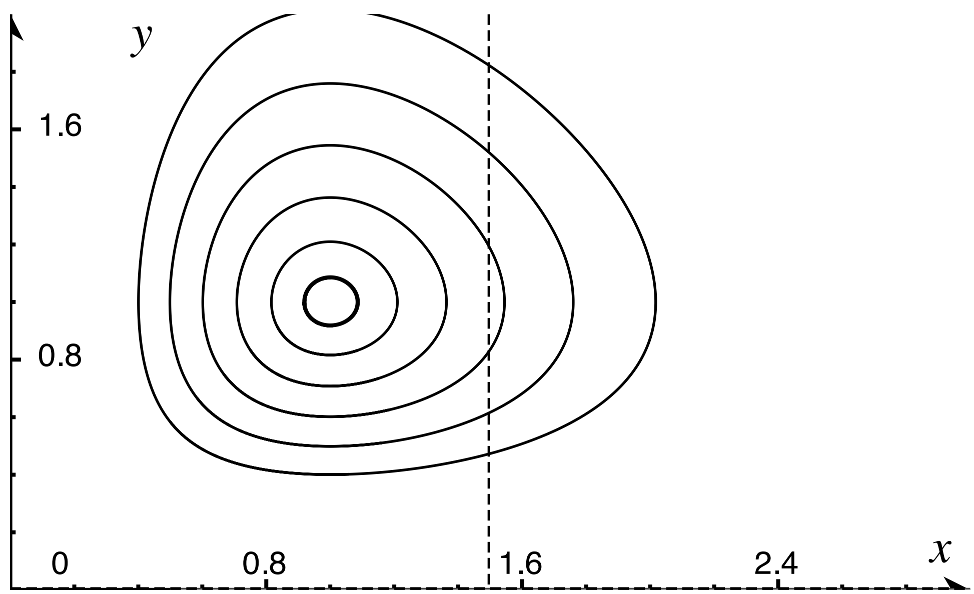
Predator-Prey Models

The first predator-prey model we considered was the Volterra model, here presented in dimensionless form:

$$\dot{N}_1 = N_1(1 - N_2),$$
$$\dot{N}_2 = \alpha N_2(N_1 - 1).$$

A phase plane for this model is graphed below.

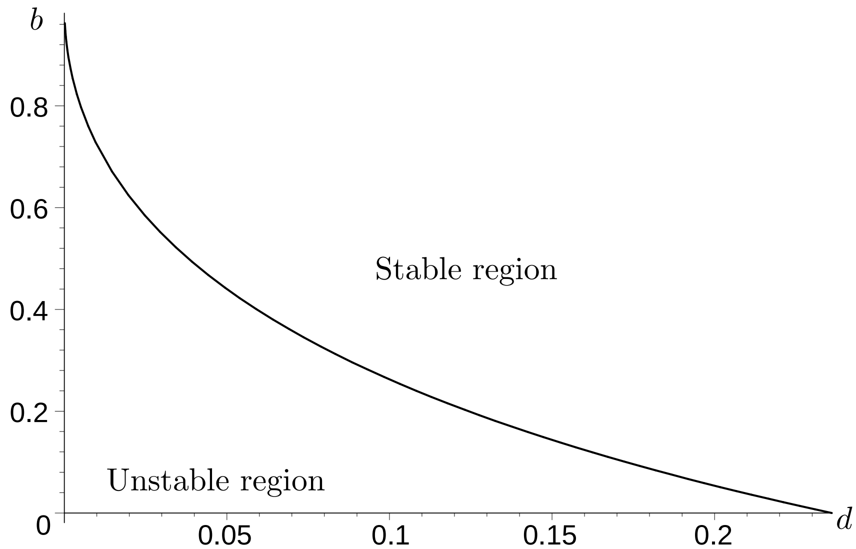


Phase plane for Volterra model, $\alpha = 1$.

Because of the lack of physical justification for the Volterra model, we tried the following new model:

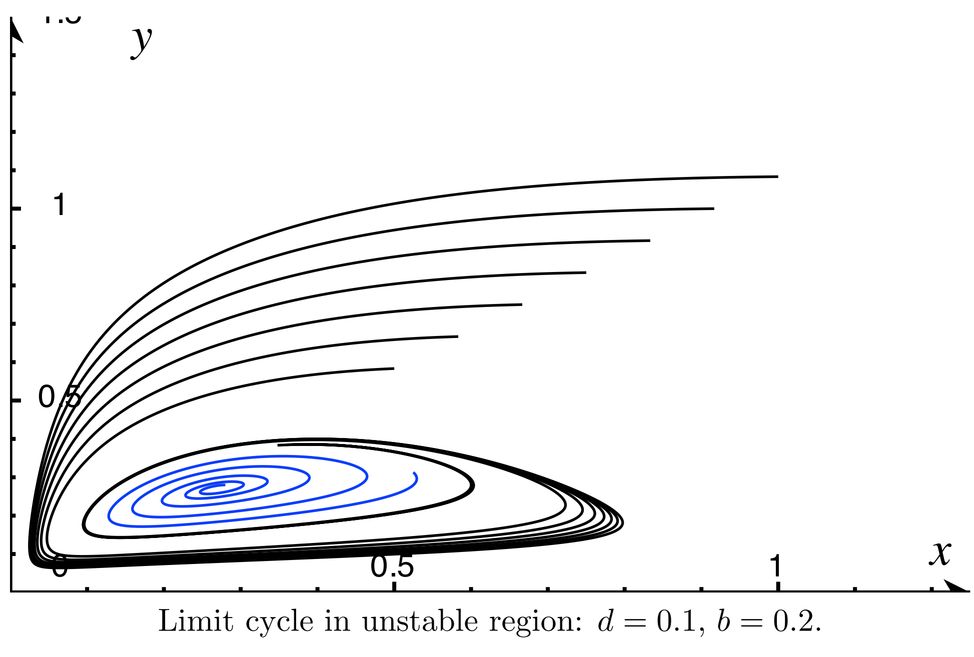
$$\dot{N}_1 = N_1 \left(1 - N_1 - \frac{N_2}{N_1 + d}\right),$$
$$\dot{N}_2 = bN_2 \left(1 - \frac{N_2}{N_1}\right).$$

We noted that the only nontrivial fixed point changed stability depending on the values of b and d, as shown in the graph below.



Stability regions for fixed point in new model.

Whenever the fixed point was unstable, we saw that the only possibility (due to inward flow from infinity) was that there must be a stable limit cycle, shown below.



Limit cycle in unstable region: $d = 0.1$, $b = 0.2$.