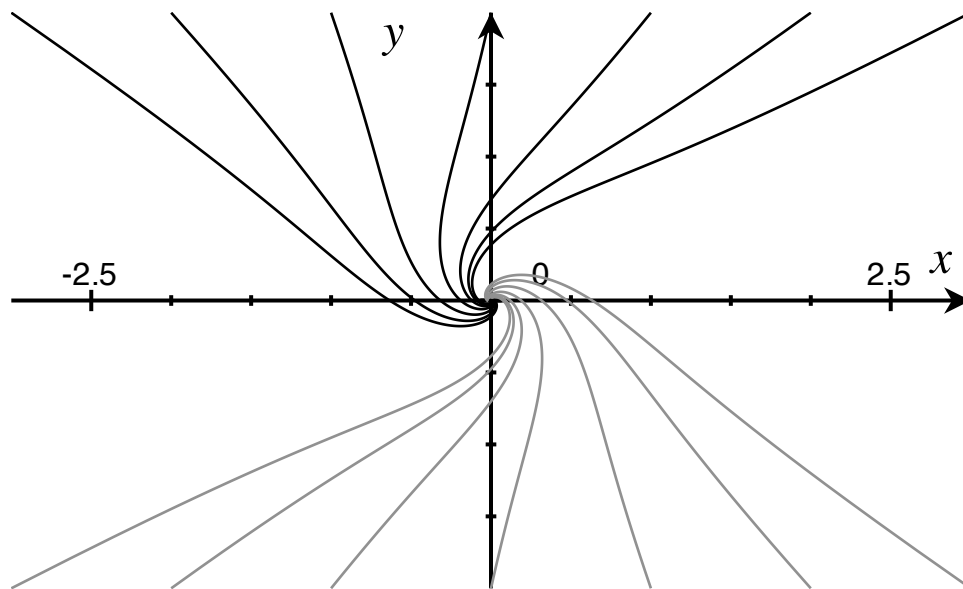


# The Hopf Bifurcation

In class, we discussed the solutions of the following system:

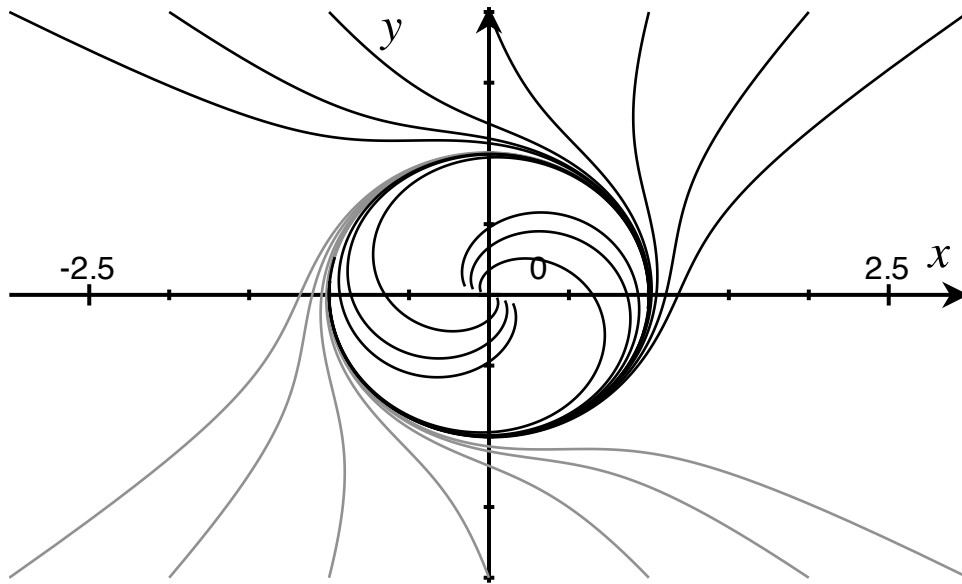
$$\begin{aligned}\dot{x} &= \alpha x - \omega y - x(x^2 + y^2), \\ \dot{y} &= \omega x + \alpha y - y(x^2 + y^2).\end{aligned}$$

We noted that the sign of  $\alpha$  determined whether there was a stable limit cycle outside the origin, and the sign of  $\omega$  determined whether the trajectories spun clockwise or counterclockwise.



$\alpha = -1, \omega = 1$ . No limit cycle; counterclockwise rotation.

This figure shows the case where  $\alpha$  is negative. In this case, the origin is a stable spiral.  $\omega$  is positive here, so the trajectories spin inward in a counterclockwise direction.



$\alpha = 1, \omega = -1$ . Limit cycle with clockwise rotation.

This figure shows the case where  $\alpha$  is positive. In this case, the origin is an unstable spiral and there is a stable limit cycle at  $r = 1$ .  $\omega$  is negative here, so the trajectories spin in a clockwise direction.