Beddington Model (Revised)

Consider the Beddington model presented in class:

\[
\begin{align*}
    x_{t+1} &= x_t \exp(r(1 - x_t) - y_t), \\
    y_{t+1} &= c(1 - e^{-y_t})x_t, \quad c = \beta aK.
\end{align*}
\]  

(3a) (3b)

The fixed point at the origin is unstable to perturbations in the host. The fixed point at \((1, 0)\) is stable for \(r < 2, c < 1\), as shown below.

Iterates of (3) for \(r = 3/2, c = 1/2\). Fixed point at \((1, 0)\) is stable.
If \( c > 1 \), there is a third fixed point given by

\[
x_* = \frac{y_*}{c(1 - e^{-y_*})}, \quad r = y_* \left[1 - \frac{y_*}{c(1 - e^{-y_*})}\right]^{-1},
\]

the coordinates of which are graphed below vs. \( r \).

Nontrivial fixed point of (3) vs. \( r \) with \( c = 2 \). Dotted curve: \( x_* \). Solid curve: \( y_* \).
Determining stability of the nontrivial fixed point with $c = 2$. Solid curve: $|\text{tr} J|$. Dashed curve: $1 + \det J$. Here $r_* \approx 3.39$. The stability is determined by the Jury conditions, as shown above.

Trajectories of (3) with $c = 2, r = 2 < r_*$. In the case where $r < r_*$, the fixed point is stable, so the populations converge to $(x_*, y_*)$. 
Trajectories of (3) with $c = 2$, $r = 3.9 > r_\ast$. $\times$ marks $(x_\ast, y_\ast)$.

In the case where $r > r_\ast$, the fixed point is unstable, so the populations oscillate about $(x_\ast, y_\ast)$. 