

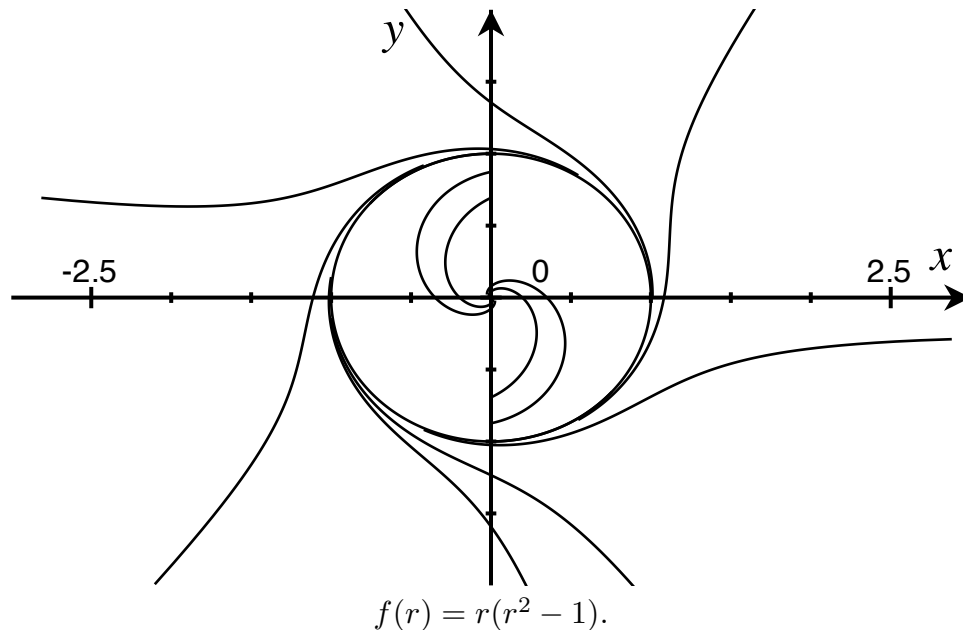
Action-Angle Coordinates

In class, we discussed the solutions of the following system in *action-angle* coordinates:

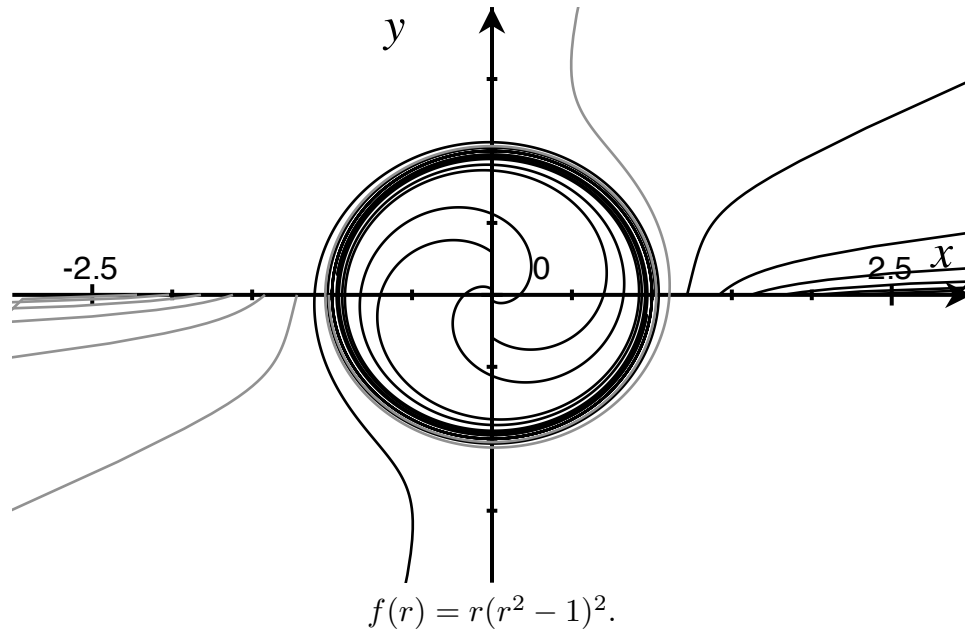
$$\begin{aligned}\dot{r} &= f(r), \\ \dot{\theta} &= \omega.\end{aligned}$$

We noted that the sign of ω determined whether the trajectories spun clockwise or counterclockwise, while the existence and stability of limit cycles could be determined from the \dot{r} equation.

In all the figures that follow, $\omega > 0$, so we have counterclockwise rotation.



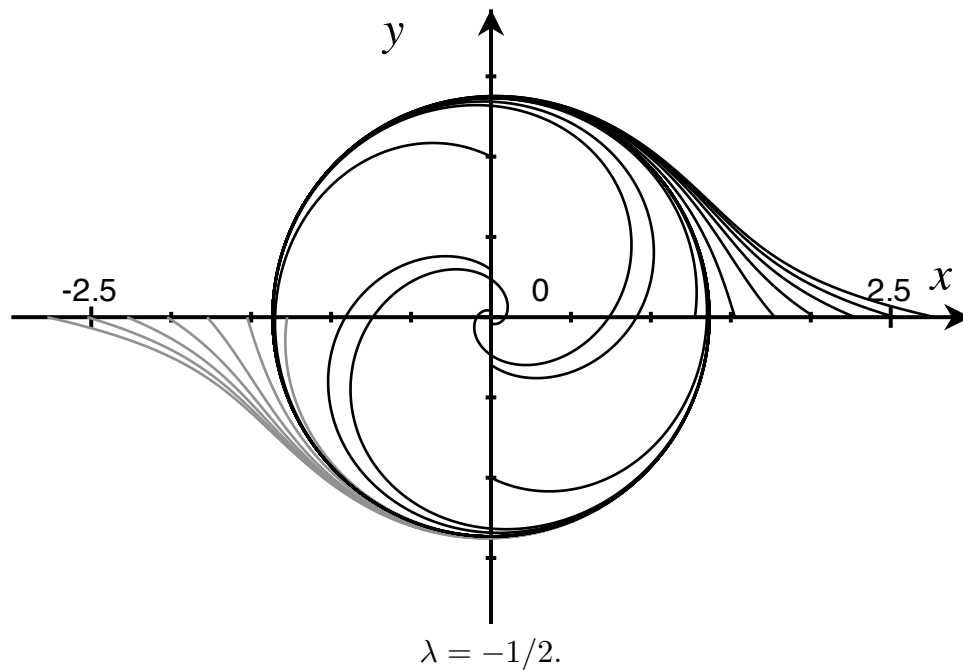
This figure shows the case where $f(r) = r(r^2 - 1)$. Here there is an unstable limit cycle at $r = 1$; this is essentially the opposite of the Hopf bifurcation diagram.



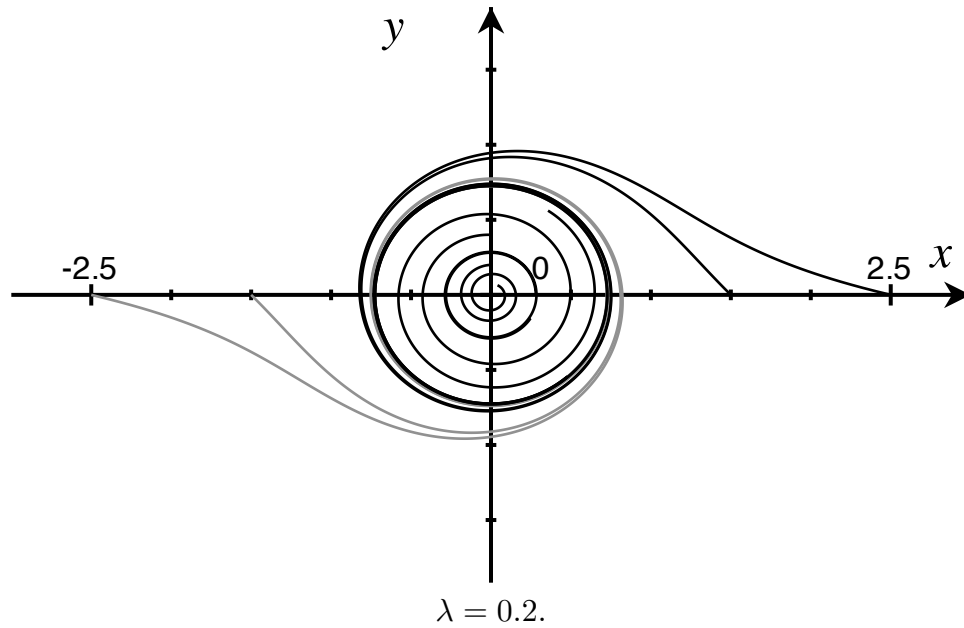
This figure shows the case where $f(r) = r(r^2 - 1)^2$. Here there is a semistable limit cycle at $r = 1$. Note the extra windings indicating a very slow approach/departure from $r = 1$.

The next three diagrams concern the case where

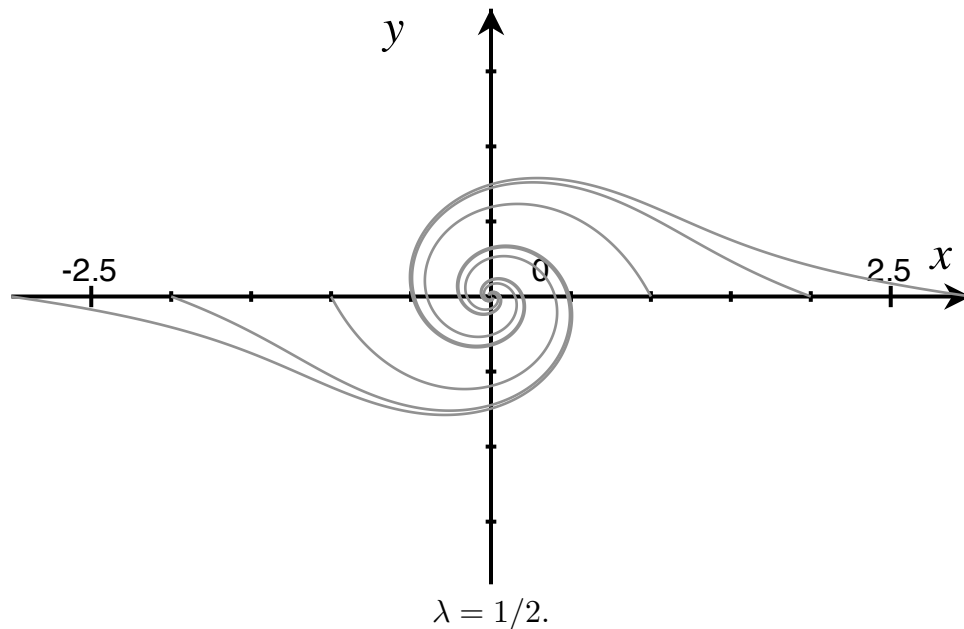
$$f(r) = -r(r^2 - r + \lambda).$$



In class we showed that if $\lambda < 0$, the origin is an unstable spiral and there is one stable limit cycle. This case is shown above.



In class we showed that if $0 < \lambda < 1/4$, the origin is a stable spiral, then moving outward one finds one unstable and one stable limit cycle. This case is shown above.



In class we showed that if $\lambda > 1/4$, the origin is stable and there are no limit cycles. This case is shown above.