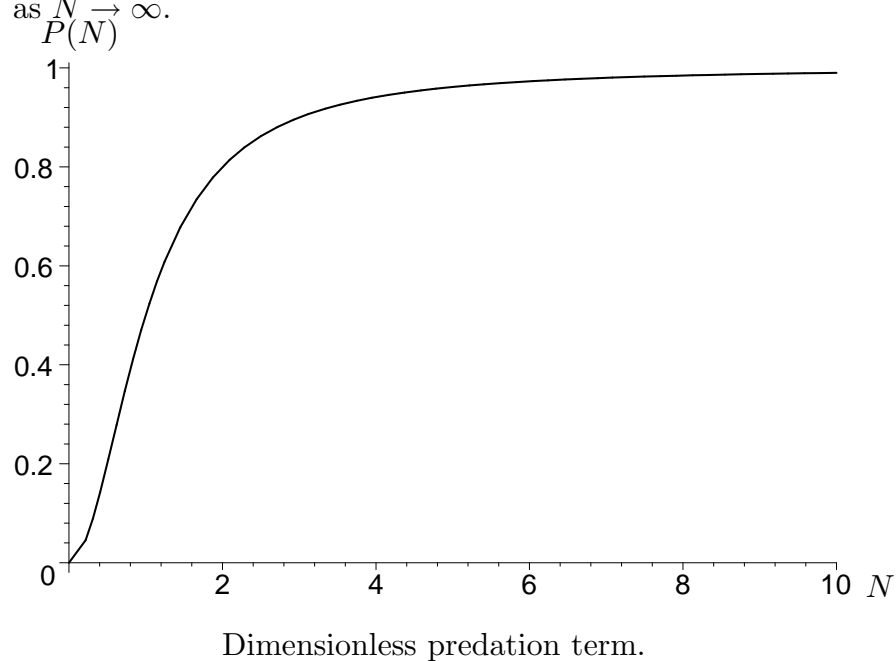


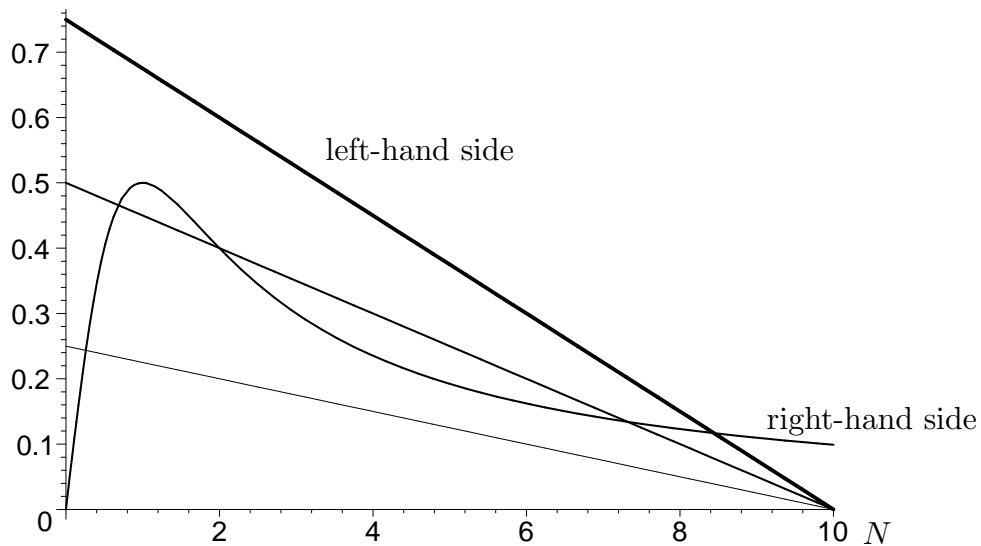
Single-Species Differential Equations

In class, we obtained the following dimensionless model for the population N of the spruce budworm:

$$\dot{N} = rN \left(1 - \frac{N}{q} \right) - P(N), \quad P(N) = \frac{N^2}{1 + N^2}.$$

Below is shown a graph of the predation term. Note that it starts of small, then saturates at the value 1 as $N \rightarrow \infty$.



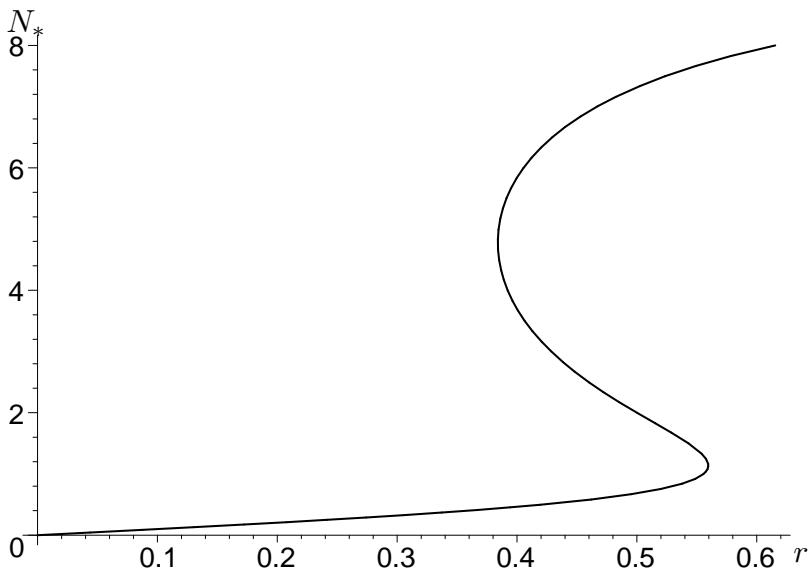


Left- and right-hand sides of (1) for $q = 10$. In increasing order of thickness: $r = 1/4$, $r = 1/2$, and $r = 3/4$.

We found that in addition to the case $N = 0$, the fixed points were given by the solutions of

$$r \left(1 - \frac{N}{q} \right) = \frac{N}{1 + N^2}. \tag{1}$$

A graph of the left- and right-hand sides of (1) is shown above. Note that as we increase r for a given value of q we have one, three, and then again one root. This is shown in the hysteresis curve for the same value of q , shown below.



Hysteresis curve for $q = 10$.