Homework Set 2

Read sections 1.3–2.3.

Single-Species Population Dynamics

1. Suppose that in the spruce budworm model we replace the predation term by

\[ \hat{P}(\bar{N}) = B \left[ 1 - e^{-((\bar{N}/N_w)^2)} \right], \]

where \( N_w = A\epsilon \) models the width of the transition region from no predation to full predation.

(a) (5 points) Use the same scalings as those given in class to scale the evolution equation which results.

(b) (5 points) Show algebraically that there is always at least one positive steady state, and illustrate graphically all possible cases.

(c) (5 points) Show algebraically that in the limit that \( \epsilon \to 0 \), for there to be three positive steady states, we must have \( qr > 4 \).

(d) (3 points) Classify each of the steady states as stable or unstable.

The Phase Plane

2. Consider the following phase-plane system:

\[ \dot{x} = -x + x^3, \quad (2.1a) \]
\[ \dot{y} = ax + y. \quad (2.1b) \]

(a) (3 points) Find the equilibrium points.

(b) (3 points) Classify each of the equilibrium points.

3. Consider the following ordinary differential equation:

\[ \frac{d^2x}{dt^2} - \frac{dx}{dt} - x(1 - \log |x|) = 0. \quad (2.2) \]

(a) (3 points) Rewrite (2.2) as a set of two coupled first-order ODEs.

(b) (3 points) Find the equilibrium points.

(c) (5 points) Describe the behavior of solutions near the equilibrium points.

(d) (5 points) Sketch the phase plane.