2-D Discrete Systems

Consider the two-dimensional system

\[ x_{t+1} = -\frac{x_t(y_t + 1)}{2}, \]
\[ y_{t+1} = \frac{y_t(1 - x_t)}{2}. \]  \hspace{1cm} (1)

Trajectories are shown below. Note that even though the origin is stable, the existence of the unstable fixed point at \((-1, -3)\) means that trajectories with slightly different initial conditions will have grossly different behavior.
Consider the two-dimensional system

\[ \begin{align*}
x_{t+1} &= \alpha y_t, \\
y_{t+1} &= -2x_t. 
\end{align*} \tag{2} \]

Trajectories are shown below for various values of \( \alpha \).

Trajectories of (2), \( \alpha = 1/6 \).

When \( \alpha < 1/2 \), the origin is stable, so this trajectory spirals in.
Trajectories of (2), $\alpha = 3/2$.

When $\alpha > 1/2$, the origin is unstable, so this trajectory spirals out.
When $\alpha = 1/2$, we have a 4-cycle.