

Separation of Variables Table (Revised)

Equation to be Solved

Function	Polar Wave	Polar Laplace	Spherical Laplace
$\Theta(\theta)$ (#1)	$\sin \nu\theta$ $\cos \nu\theta$	$\sin \lambda\theta$ $\cos \lambda\theta$	$\sin \lambda\theta$ $\cos \lambda\theta$
$R(r)$	$J_\nu(r\sqrt{\lambda})$ $Y_\nu(r\sqrt{\lambda})$	r^λ $r^{-\lambda}$ (#3)	r^ν $r^{-\nu-1}$ (#4)
$\Phi(\phi)$	_____	_____	$P_\nu^\lambda(\cos \phi)$
$T(t)$	$\sin t\sqrt{\lambda}$ $\cos t\sqrt{\lambda}$	_____	_____

Notes.

1. In this class, ν and λ will generally be integers.
2. It is recommended to start with $\Theta(\theta)$ and end with $T(t)$.
3. If $\lambda = 0$, there is also a $\log r$ term. However, it diverges both as $r \rightarrow 0$ and $r \rightarrow \infty$, so it appears only in annulus problems.
4. Note that if $\nu = 0$, $r^\nu = \text{constant}$, and this term is used whether r goes to zero or infinity.

