Updates

1. Office hours on Friday will be held from 9–10.

Homework Set 9 (Revised)

Read sections 10.3, 10.4.

The Heat Equation

1. (5 points) Find the solution of the following equation using Laplace transforms:

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \qquad x > 0, \quad t > 0, \tag{9.1a}$$

$$T(x,0) = 1, \qquad T(0,t) = H(t - t_0), \quad t_0 > 0. \tag{9.1b}$$

   Here $H(\cdot)$ is the Heaviside step function, which is the same as the unit step function $u_{t_0}(t)$.
2. (5 points) Find the solution of the following system using Laplace transforms:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \qquad x > 0, \quad t > 0, \tag{9.2a}$$

$$u(x,0) = 0, \quad \frac{\partial u}{\partial t}(x,0) = 0, \qquad u(0,t) = f(t). \tag{9.2b}$$

The Fourier Transform

3. (5 points) Solve

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^3 u}{\partial t \partial x^2}; \qquad -\infty < x < \infty, \quad t > 0 \tag{9.3a}$$

   subject to

$$u(x,0) = f(x); \qquad \frac{\partial u}{\partial t}(x,0) = g''(x), \quad g(\pm\infty) = 0 \tag{9.3b}$$

   using Fourier transforms.

4. Let

$$f(x) = e^{-ax^2} \sin bx.$$

(a) (5 points) Calculate $\hat{f}(k)$.
(b) (10 points) Use Parseval's Theorem to show that

$$\int_{-\infty}^{\infty} e^{-2ax^2} \cos 2bx \, dx = \sqrt{\frac{\pi}{2a}} \exp\left(-\frac{b^2}{2a}\right).$$

(c) (10 points) Solve the following *integral* equation for u:

$$\int_{-\infty}^{\infty} e^{-z^2/2} u(x-z) \, dz = e^{-x^2/4} \sin x.$$