MATH 535-011 — Introduction to Partial Differential Equations
Prof. D. A. Edwards — Due: Apr. 15, 2015

Homework Set 6

Read sections 5.3, 5.5, 7.7.

Polar Coordinates

1. (7 points) For the heat equation in polar coordinates:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} = \frac{\partial \psi}{\partial t}$$

   find the ordinary differential equations implied by separation of variables.

Sturm-Liouville Theory

2. (4 points) page 175, exercise 5.5.3

Bessel Functions

3. (6 points) Use the integral representation of the Bessel functions to prove that J_n is even if n is even and odd if n is odd. (*Hint: Let* $\phi = \pi - \theta$.)

The Wave Equation

4. (12 points) Solve

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u, \quad 0 < r < 1;$$

$$\frac{\partial u}{\partial r}(1, \theta, t) = 0, \quad u(r, \theta, 0) = f(r), \quad \frac{\partial u}{\partial t}(r, \theta, 0) = 0.$$

   Your answer should contain $j_{1,n}$. (*Hint: Use the identity derived in class.*)

5. Consider the wave equation in an annulus:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\psi}{\partial \theta^2}=\frac{\partial^2\psi}{\partial t^2}, \quad \epsilon \le r \le 1; \qquad \frac{\partial \psi}{\partial r}(\epsilon,\theta,t)=\psi(1,\theta,t)=0.$$

(a) (8 points) Find the appropriate r-dependent eigenfunctions for the problem, and show that the eigenvalues satisfy

$$J_m(\sqrt{\lambda})=\frac{Y_m(\sqrt{\lambda})J_m'(\epsilon\sqrt{\lambda})}{Y_m'(\epsilon\sqrt{\lambda})}. \tag{6.1}$$

Do **NOT** attempt to solve the problem.

(b) (3 points) What happens as $\epsilon \to 0$?