MATH 535-011 Introduction to Partial Differential Equations
Prof. D. A. Edwards Due: Apr. 8, 2015

Updates

1. The final will be administered from 7–10 pm on Tuesday, May 26 in our normal classroom.
2. There will be an informal review session for the final from 10–12 on Friday, May 22 in EWG 210.

Homework Set 5

Read sections 4.1, 4.2, 4.4, 4.5, 7.3, 12.3.1, 12.5.

The Wave Equation

1. (4 points per part)
   (a) page 133, exercise 4.2.1(a). You may assume that ρ_0 is constant.
   (b) Where is the string sagging the lowest? How low does it sag? Interpret your answer physically as ρ_0, L, and T_0 change.
2. (5 points per part). (*Hint: To better use the results already given in class, you may assume that* $c = 1$ *and* $L = 1$*.*)
   (a) page 142, exercise 4.4.7
   (b) page 142, exercise 4.4.8

3. Consider the wave equation:

$$c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \qquad 0 \le x \le 1 \tag{5.1a}$$

subject to the boundary and initial conditions:

$$u(0,t) = 0, \qquad u(1,t) = 1, \qquad u(x,0) = x(1-x), \qquad \frac{\partial u}{\partial t}(x,0) = \sin \pi x. \tag{5.1b}$$

(a) (3 points) Subtract off an appropriate function to obtain homogeneous boundary conditions.
(b) (8 points) Show that the solution is given by

$$u(x,t) = f(x) + \sum_{n=1}^{\infty} (\alpha_n \sin n\pi ct + \beta_n \cos n\pi ct) \sin n\pi x, \tag{5.2}$$

and determine the values of $f(x)$, α_n and β_n.

4. (11 points) Consider the wave equation for a vibrating rectangular membrane:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \qquad 0 < x < 1, \quad 0 < y < 1$$

subject to the following conditions:

$$u(x,y,0) = f(x,y), \quad \frac{\partial u}{\partial t}(x,y,0) = 0,$$

$$u(0,y,t) = u(1,y,t) = \frac{\partial u}{\partial y}(x,0,t) = \frac{\partial u}{\partial y}(x,1,t) = 0.$$

Find a series solution for this problem.